

A Deterministic Inventory Model with Two Levels of Storage for Non-Degrading Commodities, Time-Dependent Demand, And Partly Backlogged Shortages

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ABSTRACT

In a market, the cost of the variables affecting the overall cost of the inventory may not be constant. Due to the fact that the price of commodities fluctuates for a variety of reasons, including low production and strong demand, poor productivity of seasonal goods as a result of natural disasters, etc., price inflation cannot be disregarded. This paper develops a deterministic inventory model with two levels of storage for non-degrading commodities, time-dependent demand, and partly backlogged shortages. Stock is moved from RW to OW using a continuous release pattern, and it is assumed that the cost of transportation is minimal. The inventory model takes into account the inflation rate for all related expenditures. It is expected that the product is sold before its self-life term has passed. Depending on the supplier's allowed delay length, several instances are examined, and the overall profit amount is contrasted. The numerical example is provided to show the model's viability.

Keywords: Two-Warehouse Inventory Mode, Non-Deteriorating, Delay and Inflation

INTRODUCTION

A step in this approach is the conceptualization and study of a two-warehouse system. A two-warehouse inventory system offers the flexibility to accommodate both constrained and limitless storage situations. Due to the varied storage conditions in the two warehouses, the inventory of decaying objects may experience variable rates of degradation. As a result, as well as for other reasons, the holding costs may also differ. It seems sense to believe that the warehouse with the lowest holding cost in the two-warehouse inventory system has a limited capacity. According to Sarma's (1987) nomenclature, the warehouse with a limited capacity would be referred to as the "own warehouse" (abbreviated as OW), while the warehouse with an infinite capacity would be referred to as the "rented warehouse" (abbreviated as RW). Actually, a situation where there are two identical warehouses might be regarded as a single-warehouse system.

Storage is a crucial component of any logistics system in any organization. Between the production and the client is the warehouse. An inventory system with two warehouses—one with a restricted capacity and the other with an infinite capacity—with the potential for various storage conditions and holding expenses is known as a two-warehouse inventory system. The impact of perishability cannot be disregarded when evaluating the ideal inventory level in the case of inventory including perishable products such as medicines, food grains, vegetables, petroleum, etc.

The pace at which an item may spoil in a two-warehouse inventory system for perishables may change greatly due to the different storage conditions. As a result, or for other reasons, the holding cost may change. Conveniently, we identify the warehouse with limited capacity as having a lower holding cost and refer to it as an own warehouse (OW), while the other is referred to as a rental warehouse (RW). Following are some examples of how a two-warehouse system operates: If the system's ideal order level exceeds the capability of OW, RW is applied. We have a single warehouse system if the ideal level falls inside the OW's capacity and the RW is not utilized.

In this instance, we'll suppose that the leased warehouse is free of charge. The choice of the warehouse's location and design has a significant role in the venture's productivity and efficiency. The amount of storage space for goods is limited in crowded marketplaces like super markets and municipal markets, etc. Sometimes the RW's location makes it impossible to do a direct transaction. In this scenario, the provider must deliver the products from the RW to the OW, where they will then be sold.

LITERATURE REVIEW

Attri, Amit et.al. (2019). In this study, a two-warehouse inventory model for degrading goods was created, with both warehouses taken into consideration being leased spaces. For each warehouse, the demand rate varies. Stocks cause the demand for the items to climb until a certain point, after which it stabilizes. In this model, it is

presumable that the demand in the first warehouse depends on the supply, and that after the additional stock is filled in the second warehouse, the demand rate becomes constant. The pace of degradation varies between the two warehouses as a result of the various storage conditions. To further clarify the research, a sensitivity analysis and numerical example are provided. The ideal system total cost is what this paper's major goal is to ascertain.

Limansyah, Taufik et.al. (2020). The reader will learn the principles underpinning the notion of managing perishable inventory from this essay. Instead, then focusing on how to use these models, the mathematical models for perishable inventory management are highlighted. A significant chunk of the global economy is made up of perishable inventory. Almost all consumables, medications, and a wide range of other items fall under this category. Most electronic products would be included on this list as well if we consider obsolescence to be a kind of perishability. We hope that this article will provide the reader a fundamental knowledge of the problems and arithmetic underpinning these models as well as the skills to explore more reading independently.

Shrotriya, Vikas et.al. (2019). In this study, we create a combined inventory model for a producer and a consumer, and we take into account an imperfect manufacturing process with two distinct demands: an exponential demand for the producer and a triangular demand for the consumer. The model takes into consideration the buyer's right to a partly backlogged shortfall. For goods that are decaying, preservation facilities are also taken into account in this model to slow down the pace of degradation. It is presumable that the production rate is influenced by demand. This model offers a notion to lessen the pace of degradation caused by inflation. Triangular fuzzy numbers are used to reduce the model's overall cost for both sharp and fuzzy environments, and the centroid approach is used to fuzzily the total cost. For this suggested model's illustration, numerical examples and sensitivity analyses are also provided.

Bhattacharjee, Nabajyoti et.al. (2022). The inventory models for non-immediately degrading goods with stock-dependent demand are covered in this study. While the cost of sales income is assumed to be a falling linear function of time, the cost of holding is a rising function of time. This factor has improved the process of creating a mathematical model for the ideal order quantity, and the value of the total profits in relation to the key parameters is pointed out with the use of a numerical example.

Luchko, Mykhailo et.al. (2019). In this study, an optimum control strategy is presented for a class of sustainable production-inventory systems (PI). There

is modeling, analysis, and control for PI systems. In order to simulate production-inventory systems, a mixed capacity/third order PI is used to investigate dynamic techniques that are useful for maximizing profits and minimizing costs across the PI. The three most important qualities that make up an appropriate control are excellence, robustness, and stability. Using the Pontryagin maximum principle and the present value Hamiltonian method (PVH), optimal control formulation is carried out. Keywords: Production-inventory systems, Optimal control, Resilience, and Sustainability.

Assumptions and notations

Assumptions

On the basis of the following presumptions, a two-warehouse inventory model is created:

- Lead time is minuscule and replenishment rate is unlimited.
- The cost of holding is the same in both warehouses.
- The inventory system has an unlimited time horizon.
- Products from OW are only eaten after those from RW have been finished.
- The OW's storage capacity is constrained, but the RW's is infinite.
- Demands are an exponential function of time and change with time.

Notations

O_c : Ordering fees per order.

W : OW's capacity.

T : how long the replenishing cycle lasts.

M : permitted amount of time.

t_μ : the period of time before an inventory level in RW disappears.

t_r : the moment when shortages start and the inventory level in OW disappears.

h_r : The holding cost per hour in OW.

h_w : The rate of holding per minute in RW and $(h_r - h_w) > 0$. $f(t)$: Demand rate as provided by $f(t) = ae^{*t}$

where $\lambda > 0$: $b(t)$: Inventory shortfall demand rate, as reported by

$b(t) = e^{-\delta t}$ backlogging parameter where $\delta > 0$ and $0 < b(t) < 1$.

If $b(t) = 1$ or (0) The shortfalls are completely backlogged or (lost) for all t .

b_c : The price of backlogs per unit of time.

l_c : Cost of a missed chance to sell.

P_c : cost per unit of the purchased product.

S_p : Sales price per unit of the product

I_p : Interest calculated on a temporal basis.

I_e : Earned interest per unit of time.

Development of Mathematical Model

when the cycle first began at $t=0$ a lot size of Q_{max} units of inventory is added to the backed-up system. $(Q_{max} - (R + W))$ after which the remaining units are cleared in OW, $(R-W)$ is maintained in RW and W units. To see Figure-1,

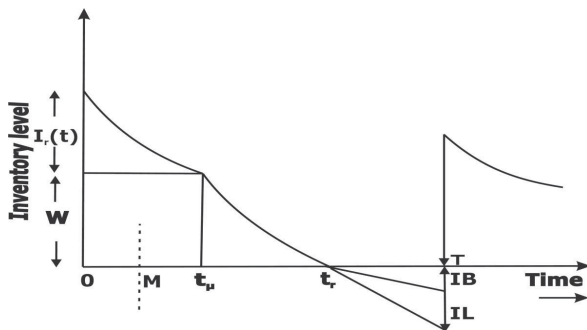


Figure 1: Graphical representation for Inventory System (Inventory Level Vs. Cycle Length)

Throughout the period $[0, t_\mu]$ Due to demand, the inventory in RW was exhausted, and at this time, the inventory level in OW is still W unit. The following differential equation governs the scenario characterizing the inventory level:

$$\frac{dI_r(t)}{dt} = -f(t); \quad 0 \leq t \leq t_\mu$$

$$\frac{dI_{w,1}(t)}{dt} = 0; \quad 0 \leq t \leq t_\mu$$

At time $t = t_\mu$, Demand is satisfied by OW until it hits zero when supply in RW runs out. During the period $[t_\mu, t_r]$, Demand causes inventory to run out in OW and is controlled by the following differential equation:

$$\frac{dI_{w,2}(t)}{dt} = -f(t); \quad t_\mu \leq t \leq t_r$$

At time $t = t_r$, When there is a lack of inventory in OW and products are consistently in demand, backlogs form at the end of the cycle length and products are delivered to consumers at the beginning of the cycle length. The scenario is described by the differential equation

$$\frac{dI_s(t)}{dt} = -b(t)f(t); \quad t_r \leq t \leq T$$

Boundary conditional solutions to equations (5.1)

to (5.4) $I_r(t) = 0$ at $t = t_\mu$, $I_{w,1}(t) = W$ at $t = 0$, $I_{w,2}(t) = 0$ at $t = t_r$ and $I_s(t) = 0$ at $t =$

T are as follows:

$$I_r(t) = \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) \quad 0 \leq t \leq t_\mu$$

$$I_{w,1}(t) = w \quad 0 \leq t \leq t_\mu$$

$$I_{w,2}(t) = \frac{a}{\lambda} (e^{\lambda t_r} - e^{\lambda t}) \quad t_\mu \leq t \leq t_r$$

$$I_s(t) = \frac{ae^{-\delta T}}{\lambda + \delta} (e^{(\lambda+\delta)t_r} - e^{(\lambda+\delta)t}) \quad t_\mu \leq t \leq T$$

At $t = t_\mu$, $I_{w,1}(t_\mu) = I_{w,2}(t_\mu)$ which results

$$t_r = f(t_\mu)$$

The present value of the cost of a missed sales opportunity is

$$I_C \left(\int_{t_r}^T (1 - b(t)f(t)) e^{-rt} dt \right)$$

Revenue Vehicle's current value is

$$S_P \{I_r(t=0) + I_{w,1}(t=0) + IB(t=T)\}$$

$$= S_P \left\{ \frac{a}{\lambda} (e^{\lambda t_\mu} - 1) + W + \frac{ae^{-\delta t}}{\lambda + \delta} (e^{(\lambda+\delta)t_r} - e^{(\lambda+\delta)T}) \right\}$$

Analyzing Various Cases

Depending on the parameters' values t_μ, t_r, M and T , the examples below are discussed:

Case-1: $0 < M \leq t_\mu < t_r < T$

Case-2: $0 < t_\mu < M \leq t_r < T$

Case-1: $0 < M \leq t_\mu < t_r < T$

Because the duration of the positive stock period exceeds the allowable delay time in this instance, the merchant must pay interest. I_p after M and as a result, interest is paid by

$$IP_1 = P_C I_P \left(\int_M^{t_\mu} \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) e^{-rt} dt + \int_{t_\mu}^{t_r} \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) e^{-rt} dt \right);$$

The merchant, on the other hand, begins to sell the item, which causes him to accrue sales and earn interest at the rate I_e on the proceeds from sales. Earned interest is provided by

$$IE_1 = S_P I_e \int_0^M \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) e^{-rt} dt + \int_{t_\mu}^{t_r} \frac{a}{\lambda} (e^{\lambda t_r} - e^{\lambda t}) e^{-rt} dt + \int_0^T \frac{ae^{-\delta T}}{\lambda + \delta} (e^{(\lambda+\delta)t_r} - e^{(\lambda+\delta)T}) e^{-rt} dt + I_e \int_M^T I_e \left(\int_0^M \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) e^{-rt} dt + \int_{t_\mu}^{t_r} \frac{a}{\lambda} (e^{\lambda t_r} - e^{\lambda t}) e^{-rt} dt \right) S_P;$$

Case-2: $0 < t_\mu < M \leq t_r < T$

Additionally, the duration of the positive stock period in this instance exceeds the allowable wait time, thus the merchant must be charged interest. I_p interest is paid by after M as a result.

$$IP_2 = P_C I_P \left(\int_M^{t_r} \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) e^{-rt} dt \right)$$

The merchant, on the other hand, begins to sell the item, which causes him to accrue sales and earn interest at the rate I_e on the proceeds from sales. Earned interest is provided by

$$IE_2 = S_P I_e \left(\int_0^{t_\mu} \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) dt \right) + \int_{t_\mu}^M \frac{a}{\lambda} (e^{\lambda t_r} - e^{\lambda t}) e^{-rt} dt + \int_M^{t_r} \frac{a}{\lambda} (e^{\lambda t_r} - e^{\lambda t}) e^{-rt} dt + \int_0^T \frac{ae^{-\delta T}}{\lambda + \delta} (e^{(\lambda+\delta)t_r} - e^{(\lambda+\delta)T}) e^{-rt} dt + I_e \int_M^T I_e \left(\int_0^{t_\mu} \frac{a}{\lambda} (e^{\lambda t_\mu} - e^{\lambda t}) dt + \int_{t_\mu}^M \frac{a}{\lambda} (e^{\lambda t_r} - e^{\lambda t}) e^{-rt} dt \right) S_P + I_e \int_{t_r}^T S_P \left(\int_M^{t_r} \frac{a}{\lambda} (e^{\lambda t_r} - e^{\lambda t}) e^{-rt} dt \right)$$

Optimality Condition

$$\frac{\partial \Pi^i(t_\mu, T)}{\partial t_\mu} = 0; \quad \frac{\partial \Pi^i(t_\mu, T)}{\partial t_\mu};$$

Provided

$$\frac{\partial^2 \Pi^i(t_\mu, T)}{\partial t_\mu^2} \frac{\partial^2 \Pi^i(t_\mu, T)}{\partial T^2} - \frac{\partial^2 \Pi^i(t_\mu, T)}{\partial t_\mu \partial T} > 0 \text{ and}$$

$$\frac{\partial^2 \Pi^i(t_\mu, T)}{\partial t_\mu^2} < 0; \quad \frac{\partial^2 \Pi^i(t_\mu, T)}{\partial T^2} < 0 \text{ at } (t_\mu, T)$$

Using the MATHEMATICA 9.0 program to solve equation (5.19), choice variable values t_μ, t_r, T being restrained $0 < t_\mu < t_r < T$ are acquired and put to use in eq. (5.18), which produces the best answer for each situation. Depending on the value of the permitted delay time M , the scenario with the highest profit is chosen as the best one.

The following instances, where the parameter values are chosen at random, are taken into consideration to verify the constructed model.

Example-5.1: Let $a = 100, \lambda = 3.0, W = 100, C_o = 100, h_r = 0.6, h_w = 0.3, b_c = 3.0, l_c = 15, S_p = 15, P_c = 10, I_p = 0.015, I_e = 0.012, r = 0.25, \delta = 0.7$. in the proper unit and $M = \frac{1}{12}, \frac{1}{4}, \frac{1}{2}$ Table -1 displays the computational outcomes for the given year.

Example-5.2: Let $a = 180, \lambda = 2.8, W = 100, C_o = 100, h_r = 0.6, h_w = 0.3, b_c = 3.0, l_c = 15, S_p = 15, P_c = 10, I_p = 0.015, I_e = 0.012, r = 0.20, \delta = 0.7$ in the proper unit and $M = \frac{1}{12}$ year. Table-2 displays the computational results.

Table-1: Representing value of decision variables and profit function

| Case | M=1/12 | | | |
|------|-------------|-----------|--------|-------------------------|
| | t_{μ^*} | t_{r^*} | T_s | $\Pi^l(t_{\mu^*}, T_s)$ |
| 1 | 2.8352 | 9.4258 | 9.5527 | 1084.79 |
| 2 | 2.6582 | 9.4536 | 9.5807 | 1207.87 |
| 3 | 3.0071 | 9.5488 | 9.6392 | 1291.02 |
| 4 | 2.8046 | 9.5642 | 9.6580 | 1364.13 |

Table-2: Representing value of decision variables and profit function

| Case | M=1/12 | | | |
|------|-------------|-----------|---------|-------------------------|
| | t_{μ^*} | t_{r^*} | T_s | $\Pi^l(t_{\mu^*}, T_s)$ |
| 1 | 4.0501 | 11.7635 | 11.8208 | 2416.63 |
| 2 | 3.8552 | 11.7952 | 11.8506 | 2646.12 |
| 3 | 4.2587 | 11.9145 | 11.9296 | 2864.47 |
| 4 | 4.0448 | 11.9392 | 11.9495 | 3005.05 |

Based on the aforementioned findings shown in Tables 1 and 2, Figures 2a, 2b, 2c, and 2d use 3-D graphical representation to show the concavity of the profit function in each scenario.

Observations

The following observations are made:

Table-1 shows that in cases 1 and 2, profit and ordering cycle duration both grow as the permitted wait time increases. In cases 3 and 4, as the permitted wait time increases, the profit and ordering cycle duration both drop. The profit is maximum in instance 4 when the permitted delay duration exceeds the ordering cycle length for each value of M, as can also be shown from Tables 1 & 2.

Sensitivity Analysis

In order to analyze the impact of changing the value of the parameters on the best policy, sensitivity analysis is carried out (for 50% values of the parameters), and the findings are shown in Table-3.

Table-3: Representing Sensitivity Analysis with respect to parameters to study change in cycle length and profit function

| Parameter | Parameter change value | t_{μ^*} | t_{r^*} | T_s | $\Pi^l(t_{\mu^*}, T_s)$ |
|-----------|------------------------|-------------|-----------|--------|-------------------------|
| a | 50 | 2.8064 | 9.4319 | 9.5416 | 617.85 |
| (100) | 150 | 2.8448 | 9.4238 | 9.5564 | 1551.70 |
| b_c | 1.5 | 2.6029 | 9.0956 | 9.2319 | 1111.01 |
| (3) | 4.5 | 3.0803 | 9.7740 | 9.8916 | 1058.90 |
| l_c | 7.5 | 2.1406 | 8.4255 | 8.6183 | 1166.39 |
| (15) | 22.5 | 3.4062 | 10.241 | 10.335 | 1027.65 |
| S_p | 7.5 | 3.6824 | 9.1826 | 9.3568 | 86.32 |
| (15) | 22.5 | 2.6105 | 9.6590 | 9.7247 | 2136.41 |
| P_c | 5 | 3.5763 | 11.909 | 11.893 | 1598.43 |
| (10) | 15 | 2.6934 | 8.3906 | 8.5879 | 781.62 |
| h_r | 0.3 | 2.9339 | 9.4382 | 9.5574 | 1098.93 |
| (0.6) | 0.9 | 2.7430 | 9.4142 | 9.5499 | 1071.55 |
| h_w | 0.15 | 2.9127 | 9.4441 | 9.5605 | 1118.89 |
| (0.3) | 0.45 | 2.7537 | 9.4073 | 9.5451 | 1051.32 |
| W | 50 | 2.8497 | 9.4232 | 9.5578 | 1016.57 |
| (100) | 150 | 2.8202 | 9.4284 | 9.5476 | 1152.99 |
| r | 0.125 | 7.0615 | 18.562 | 18.492 | 1868.56 |
| (0.25) | 0.375 | 1.4789 | 6.3796 | 6.5765 | 884.90 |
| l_p | 0.0075 | 2.8210 | 9.4484 | 9.5669 | 1139.81 |
| (0.015) | 0.0225 | 2.8486 | 9.4031 | 9.5386 | 1030.01 |
| l_e | 0.006 | 2.8616 | 9.4663 | 9.5923 | 1080.98 |
| (0.012) | 0.018 | 2.8101 | 9.3858 | 9.5135 | 1088.88 |
| C_o | 50 | 2.8351 | 9.4254 | 9.5532 | 1077.52 |
| (100) | 150 | 2.8353 | 9.4262 | 9.5523 | 1092.05 |
| M | 1/24 | 2.8336 | 9.4256 | 9.5527 | 1084.38 |
| (1/12) | 1/8 | 2.8366 | 9.4260 | 9.5528 | 1085.19 |
| λ | 2.4 | 3.2877 | 9.5822 | 9.6346 | 1465.46 |
| (3.0) | 4.5 | 1.8117 | 9.2016 | 9.4571 | 633.42 |
| δ | 0.35 | 2.1406 | 8.4255 | 8.6183 | 1166.39 |
| (0.7) | 1.05 | 3.4062 | 10.240 | 10.334 | 1027.65 |

Noting that $\delta=1.5$ (-50%) is impractical, we did sensitivity analysis for a value of -20%.

Observations on sensitivity

We may draw the following conclusions from Table 3:

1. The demand parameter has a significant impact on the profit function., a, λ in direct proportion to each of these variables is the selling price.
2. The profit function is directly proportional to the backlog parameter, the inflation rate, and the purchase cost, and it is very sensitive to all three of these parameters, i.e., the profit declines as these three parameters rise.
3. The parameters have a modest impact on how the profit function behaves. h_w, I_p & W and directly proportionate to all of these factors, meaning that profit rises as the value of these factors rises and falls as these factors fall.
4. The profit function is indirectly related to the opportunity cost of missed opportunities and the backlog cost.
5. The profit function is directly proportional to other factors and is only marginally sensitive to them.

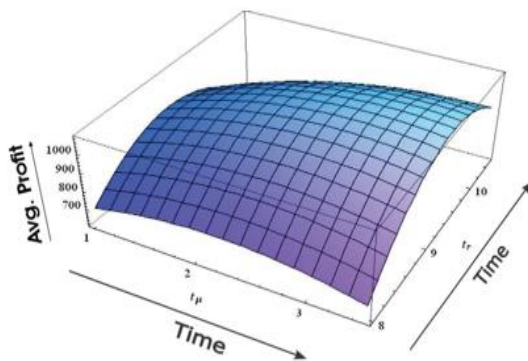


Figure-2a: Graphical representation of cost function depicting concavity w.r.t t_μ and t_r for Inventory System (Case-1)

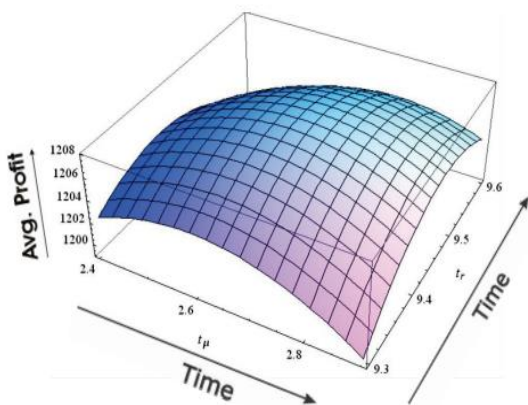


Figure-2b: Graphical representation of cost function depicting concavity w.r.t t_μ and t_r for Inventory System (Case-2)

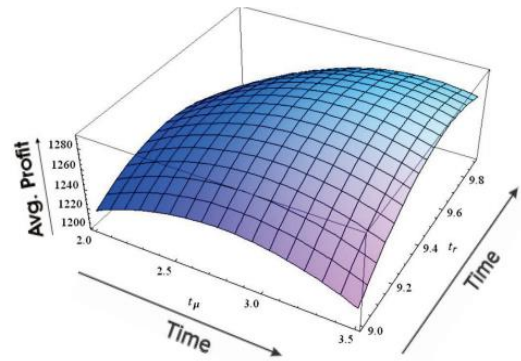


Figure-2c: Graphical representation of cost function depicting concavity w.r.t t_μ and t_r for Inventory System (Case-3)

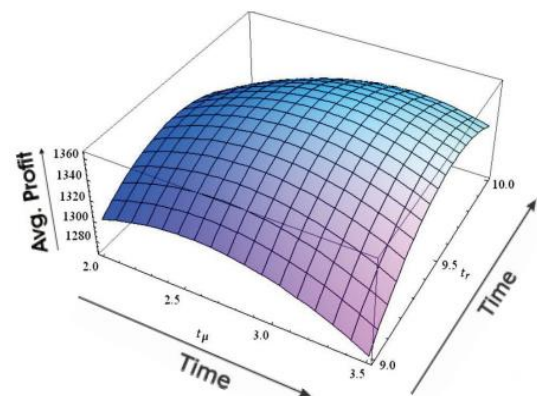


Figure-2d: Graphical representation of cost function depicting concavity w.r.t t_μ and t_r for Inventory System (Case-4)

CONCLUSION

In order to maximize the profit function of the inventory system, a deterministic two-warehouse inventory model for non-degrading products with exponential demand and allowable payment delay under inflation is constructed in this paper. Shortages are permitted, and the backlog is growing exponentially. Profit, it has been shown, is influenced by pricing, demand, and inflation. The suggested model may be utilized for inventory management of certain non-degrading goods, including electrical parts for appliances, televisions, machineries, plastic toys, and other goods.

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