Journal of Harbin Engineering University
ISSN: 1006-7043

Vol 45 No. 1
January 2024

Statistical and Instantaneous CSI based Outage Performance Analysis for DL-NOMA

Najuk Parekh[1], Dr. Rutvij Joshi[2], Dr. Twinkle Doshi[3]
[1] Research Scholar, Gujarat Technological University, Gujarat, India
[2.] Electrical Engineering Department, Sardar Patel College of Engineering, Bakrol, Gujarat, India
[3] Computer Application Department, Faculty of Science, The Maharaja Sayajirao University of Baroda., Vadodara, India

Abstract

In 5G and beyond, Non-Orthogonal Multiple Access (NOMA) can provide a promising alternative for multiple access. An arbitrary number of users on a single carrier multi-user NOMA DL system is analysed using closed-form outage probabilities in this paper. User order has been obtained for statistical CSI assumed at the base station and Instantaneous CSI assumed at both base station and receiver. Both user-ordering schemes are examined extensively for different data rates targeted at a given fixed SNR for individual users' QoS requirements. The system model is tested by simulations, which reveal that the Instantaneous CSI-based user ordering scheme yields superior results over the Statistical CSI-based user ordering scheme.

Keywords: 5G, Instantaneous CSI, Multiple Access Technique, NOMA, Outage Analysis, Statistical CSI.

1. Introduction

A 5G network needs to be highly spectral efficient, with low latency, as well as massively connected [1]. There should be a robust network that can support all four major applications, such as enhanced mobile broadband (eMBB), massive machine-type communications (mMTC), ultra reliable and low latency communications (URLLC), and enhanced vehicle-to-everything connectivity (eV2X).

A NOMA system works as a combination of SC on the transmitter side and SIC on the receiver side. When signals are transmitted from the transmitter side it uses the concept of power allocation in which users are scheduled/ multiplexed according to their channel condition with a power allocation coefficient to maximize system performance. The NOMA system's performance was extensively assessed through the analysis of outages.

In previous papers, it carries various problems such as resource allocation, multicarrier NOMA technology, user scheduling, and user selection/ grouping. This paper focuses on two CSI techniques which are based on the ordering of users, statistical and an instantaneous CSI-based user ordering methods.

Apart from other papers, a more realistic model is taken by including distance and path loss components as a PDF variable and on which the average channel gain is calculated and accordingly the powers are assigned. Li et al. in [2] investigate multi-carrier NOMA with closed outage analysis for an arbitrary number of users. It is based on statistical CSI at the transmitter.

According to Ding et al., "Joint user grouping and decoding order uplink/downlink MISO/SIMO-NOMA" can be achieved. In which outage depends on the target rates and location of nodes, and the results, it is possible to maximize outage by adjusting power allocation factors. With Nakagami-m fading channels and statistical CSI, the closed-form expression of the outage has been carried out in terms of systems sum throughput with power allocation for the different users, in which pairing of users is considered with Genetic Algorithm, the author has addressed 2 user case with different power allocation coefficients.

Another CDF-based scheduling over Nakagami-m fading channel was carried out in [4], where users are selected based on the CS algorithm, and an outage has been carried out on that selected user's targeted data rate which has been taken as 1.2 and 2 bps/Hz for the two-user scenario. In his study, the author found that outage performance depended on the number of users and the Nakagami-m fading parameter. In [5] the rate regions have been found out for outage probability constraints, and a new SIC ordering has been proposed. In his study, the author found that outage performance depended on the number of users and the Nakagami-m fading parameter.

An analysis of outage performance in an i.ni.d environment has been discussed in [6]. Dynamic SIC ordering has been considered, and the outage is expressed for an arbitrary number of users. Conditional outage probability according to decoding order is proposed as a joint probability with linear constraints on decoding order and required data rate.

Tweed et al. [7] have addressed resource allocation for Uplink NOMA and carried out outage constraints for critical applications with imperfect SIC errors variance. The outage probabilities and their asymptotic approximations at high SNR for a two-user case using a dynamic power scheme based on hybrid NOMA are given in [8] for fixed NOMA, CR-NOMA, and DH-NOMA.

The key novelty in this work is to derive the closed-form expression of individual outage analysis of users in terms of their knowledge of the statistical CSI. As a result, the most significant part of our work has been devoted to the analysis and comparison of the NOMA DL system both in statistical CSI-based user ordering and in Instantaneous CSI-based user ordering. The analysis and comparison are done with individual QoS requirements for the users. A threshold data requirement is set on the basis of the number of users (i.e. 2, 3, or 4) and on the basis of which an application can be determined.

2. System Model

A Single Carrier Non-Orthogonal Downlink system with one base station and K number of users equipped with one antenna is considered here. The channel is assumed to be slow fading [9], and CSI is perfectly available at the receiver. A PDF of a Rayleigh fading channel contains the channel fading coefficient h_i, with the components of small-scale fading and path loss based on the distance between the base station and the user k_i, $i \epsilon \{1,2,\dots,K\}$, is given as [10],[11]

$$f_{|H_k|^2}(|h_k|^2) = \frac{1}{\delta_K^2} e^{-\frac{|h_k|^2}{\delta_K^2}} \qquad (1)$$

Here δ_i is average channel gain which is modelled as $\delta_i^2 = \chi_i {dm_i}^{-\lambda}$, χ_i denotes the squared magnitude of channel co-efficient which indicates small scale fading giving, $\mathbb{E}[{\chi_i}^2] = 1$, λ is a path loss component, dm_i is a distance of i-th users from the base station. The users' distances are $dm_1 > dm_2 > \cdots > dm_K$ in meters.

Receiver-side SIC detection is also performed on the signals using channel gain information [2], and interference cancellation are assumed to be perfect. Initially, the strongest signal is decoded by treating all other signals as interference, then the second strongest signal is decoded from the composite signal and cancelled out, and so on. The process is repeated until the weakest signal is detected. This SIC detection order is also influenced by the ordered channel gain information [2]. Therefore, a separate analysis was conducted for statistical CSI and instantaneous CSI for SC-NOMA DL systems.

Within this system, the fixed power allocation (FPA) scheme is examined with a fixed power allocation co-efficient, where all the co-efficient add up to one. Details on allocated power are given in the following subsection.

A. Statistical CSI based Ordering for NOMA DL

A channel gain average from the base station to the end-user k_i is ordered as $\delta_1 \le \delta_2 \le \cdots \le \delta_K$. Base station users have access only to statistical CSI information, knowing the perfect CSI. δ_i is average channel gain, which is given by $\delta_i = \mathbb{E}(|h_i|^2)$.

Now, in DL NOMA the base station linearly combines all users' symbols as superposed signals with differently allocated power as $a_1 \ge a_2 \ge \cdots \ge a_K$. Because the NOMA DL system assigns greater power to the user farthest away from the base station, this user experiences the worst channel condition when compared to the user nearer the base station. The summation of all power co-efficient is always one i.e. $\sum_{i=1}^{K} a_i = 1$. The superposed signal is given as

$$S_s = \sum_{i=1}^{K} \sqrt{a_i P_s}\, S_i \qquad (2)$$

It is transmitted by the base station to all users in the DL NOMA system over the same radio resource block, and is received by each individual i-th user as a

$$y_i = h_i S_s + n_i \qquad (3)$$

S_s is the superimposed signal which has S_i symbols of each user, h_i follows Rayleigh faded distribution, n_i represents i.i.d additive white Gaussian noise which belongs to $\mathcal{CN}(0, \sigma^2)$, where σ^2 denotes noise power and P_s is transmitted power.

The receiver performs SIC to extract the data signal from the received signal y_i, in our case, the signal at U_1 has coherent detection of its own signal, which is also the strongest signal. In this system U_1 (stronger signal) is detected and subsequently removed, while other signals are considered as interference. This is followed by a re-modulation and cancellation of the decoded signal y_i to have the signal at a particular user. Thus, the signal at the receiving end U_i removes all the signals related to the other users which are U_2, U_3,…, U_{i-1} from the received signal, while other signals U_{i+1}, U_{i+2}, …, U_K are considered as noise. Here's the signal-to-interference-plus-noise-noise ratio (SINR) for U_K as follows:

$$SINR_K = \begin{cases} \frac{|h_i|^2 a_j \rho_s}{1+|h_i|^2 \rho_s \sum_{k=j+1}^{K} a_k} & i \le K, j < K \\ |h_K|^2 a_K \rho_s & i = j = K \end{cases}$$

(4)

Here,$\rho_s = \frac{P_S}{\sigma^2}$ is the total transmit SNR. ,a_j and a_k are allocated fixed power co-efficient related to user U_j and U_k respectively.

B. Instantaneous CSI based Ordering for NOMA DL

Although CSIs at the base station and at the user are identical, here, user channel statistics are not identical. In order to determine which users are ordered based on their instantaneous Rayleigh channel fading power as $|\widehat{w_1}|^2 \le |\widehat{w_2}|^2 \le \cdots \le |\widehat{w_K}|^2$, where,

$$|\widehat{w_1}|^2 = min(|h_1|^2, |h_2|^2, \dots, |h_K|^2) \quad (5)$$

$$|\widehat{w_K}|^2 = max(|h_1|^2, |h_2|^2, \dots, |h_K|^2) \quad (6)$$

The signal received i-th ordered user $\widehat{U_i}$ can be

$$\widehat{y_\iota} = \widehat{w_\iota}\widehat{S_s} + \widehat{n_\iota} \quad (7)$$

Here, $\widehat{S_s} = \sum_{i=1}^{K} \sqrt{\widehat{a_\iota} P_S}\, \widehat{S_\iota}$ is a superposed signal with $\widehat{S_\iota}$ symbols of each user. The power factor $\widehat{a_\iota}$ follows the opposite order based on the NOMA principle which is $\widehat{a_1} \ge \widehat{a_2} \ge \cdots \ge \widehat{a_K}$. The SIC is performed with Instantaneous CSI ordering, now the SINR and SNR for $\widehat{U_J}$ and $\widehat{U_k}$ are given respectively as

$$\widehat{SINR}_i = \begin{cases} \frac{|\widehat{w_\iota}|^2 \widehat{a_J} \widehat{\rho_S}}{1+|\widehat{w_\iota}|^2 \widehat{\rho_S} \sum_{k=j+1}^{K} \widehat{a_k}} & i \le K, j < K \\ |\widehat{w_K}|^2 \widehat{a_K} \widehat{\rho_s} & i = j = K \end{cases}$$

(8)

Here $|\widehat{w_\iota}|^2$ is instantaneous Rayleigh channel fading power, $\widehat{\rho_s} = \frac{P_S}{\sigma^2}$ is total transmit SNR, and $\widehat{a_J}$ and $\widehat{a_k}$ are instantaneous fixed allocated powers to the users $\widehat{U_J}$ and $\widehat{\mathrm{U}}_k$ respectively.

3. Outage Probability Analysis with Fixed Power Allocation

The performance of the SC-NOMA DL system with statistical and instantaneous CSI ordering is discussed in this section. We have assumed perfect interference cancellation at the receiver and the user links are i.i.d Rayleigh faded, and the PDF of which is given by Equation (1). A Fixed Power Allocation Coefficient (FPA) has been used to analyse outage probability. Let the two users $U_i, 1 \le i \le K$ and $U_j, j \le i$, and their allocated powers are a_i , a_j respectively, where $a_i + a_j = 1$ and $a_i > a_j$. The analysis has been carried out for both user ordering schemes.

The achievable data rates of each user are expressed, based on equations (4) and (8) as for statistical and instantaneous CSI is,

$$R_i = log_2\left(1 + \frac{|h_i|^2 a_j \rho_s}{1+|h_i|^2 \rho_s \sum_{k=j+1}^{K} a_k}\right), i \le K, j < K$$

(9)

$$R_j = log_2(1 + |h_K|^2 a_K \rho_s), i = j = K \quad (10)$$

$$\widehat{R_\iota} = log_2\left(1 + \frac{|\widehat{w_\iota}|^2 \widehat{a_J} \widehat{\rho_S}}{1+|\widehat{w_\iota}|^2 \widehat{\rho_S} \sum_{k=j+1}^{K} \widehat{a_k}}\right), i \le K, j < K$$

(11)

$$\widehat{R_J} = log_2(1 + |\widehat{w_K}|^2 \widehat{a_K} \widehat{\rho_s}\,), i = j = K \quad (12)$$

Where,R_i, R_j, are Statistical based CSI user ordering scheme's achievable sum rate, and $\widehat{R_\iota}$, $\widehat{R_J}$ are Instantaneous based CSI user ordering scheme's achievable sum rate.

A. Outage Analysis for Statistical CSI based Ordering for users U_i and U_j

Here, outage probability[5] is given as " Given $\widetilde{R_\iota}$ (Desired data Rate), h_i (Rayleigh Fading channel co-efficient), δ_i (channel gain), a_i (power allocation coefficient) and statistical CSI which is available at base-station, the outage probability for $the\ U_K$, denoted as P_{OUT}^K is defined as "Probability of fail-to-attain the desired data rate by user K under a given decoding order $\delta_1 \le \delta_2 \le \cdots \le \delta_K$."Based on the derived closed-form formulation we analysed users' different QoS requirements and system throughput.

Now, according to the definition, U_K's outage probability for Statistical CSI NOMA DL can be represented as

$$P_{OUT}^{K} = Pr\left\{\begin{matrix}\left(C_{U_1} < \widetilde{R_1}\right) \cup \left(C_{U_2} < \widetilde{R_2}\right) \cup \dots \\ \cup \left(C_{U_K} < \widetilde{R_K}\right)\end{matrix}\right\}$$

(13)

Where, $C_{U_K} = log_2(1 + SINR_K)$, $\widetilde{R_K}$ defines U_K's desired data rate.

If we consider two user scenario where $\widetilde{R_\iota}$ and $\widetilde{R_J}$ denotes the desired data rates of U_i and U_j when i-th and j-th user are chosen, then according to the definition, the condition for the outage for user U_i is given as

$$A_i \equiv \begin{matrix}\left\{log_2\left(1+\frac{|h_i|^2 a_j \rho_s}{1+|h_i|^2 \rho_s \sum_{k=j+1}^{K} a_k}\right)<\widetilde{R_\iota}\right\} \\ P\left\{|h_i|^2<\frac{X_i}{\left(a_i-\left(\sum_{k=j+1}^{K} a_k X_i\right)\rho_s\right)}\right\}\end{matrix}$$

(14)

Here, $X_i = 2^{\widetilde{R_\iota}} - 1$ which satisfies the constraint $X_i < \frac{a_j}{\sum_{k=j+1}^{K} a_k}$, $|h_i|^2$and $\left|h_j\right|^2$are channel gains of U_i and U_j, with Rayleigh faded and $E\{|h_i|^2\} = {\delta_i}^2$and PDF of which is given as in equation no (1). A_i is the condition of outage probability of U_i, pursuant to which an outage occurs when the maximum transmission rate at which the data U_j can successively decode by U_i with interference from other signals i.e. $U_{j+1}, \dots, U_K$, which is more than or equal to the desired QoS ($\widetilde{R_\iota} \frac{bps}{Hz}$).

Now, the condition for outage for j-th user will be defined as

$$A_j = \begin{cases} log_2\left(1+\frac{\left|h_j\right|^2 a_i \rho_s}{1+\left|h_j\right|^2 \rho_s \sum_{k=j+1}^{K} a_k}\right)<\widetilde{R_\iota} & j<K \\ log_2\left(1+\left|h_j\right|^2 \rho_s \sum_{k=j+1}^{K} a_k\right)<\widetilde{R_J} & j=k \end{cases}$$

(15)

In the case of the j-th user, the outage probability is defined as

$$P\left\{\left|h_j\right|^2 < max\left\{\frac{X_i}{\left(a_i-X_j \sum_{k=j+1}^{K} a_k\right)\rho_s}, \frac{X_j}{a_j \rho_s}\right\}\right\}$$

(16)

Where, $X_j = 2^{\widetilde{R_J}} - 1$. The expression for outage for U_i and U_j can be given as

$$P_{OUT}^{U_i} = \int_0^{\frac{X_i}{\left(a_i-\left(\sum_{k=j+1}^{K} a_k X_i\right)\rho_s\right)}} f_{|H_i|^2}\left(|h_i|^2\right) d\left(|H_i|^2\right)$$

(17)

$$P_{OUT}^{U_j} = \int_0^{max\left\{\frac{X_i}{\left(\left(a_i-X_j \sum_{k=j+1}^{K} a_k\right)\rho_s\right.}, \frac{X_j}{a_j \rho_s}\right\}} f_{\left|H_j\right|^2}\left(\left|h_j\right|^2\right) d\left(\left|H_j\right|^2\right)$$

(18)

The Eq. (17) and (18) shows individual outage probability analysis for statistical CSI-based ordering.

C. **Outage Analysis for Instantaneous CSI based Ordering for users $\widehat{U_\iota}$ and $\widehat{U_J}$**

The outage probability is given as "Given $\widehat{R_\iota}$ (desired data rate), $\widehat{h_\iota}$ (Rayleigh Channel fading channel co-efficient), $\widehat{w_\iota}$ (instantaneous Rayleigh channel fading power) and Instantaneous CSI which is available at base-station as well at all users, the outage probability for U_k, denoted as $\widehat{P_{OUT}}^{k}$ is defined as " Probability of fail-to-attain the desired data rate by user K under a given decoding order $|\widehat{w_1}|^2 \leq |\widehat{w_2}|^2 \leq \dots \leq |\widehat{w_K}|^2$, where $|\widehat{w_1}|^2 = min(|h_1|^2, |h_2|^2, \dots, |h_K|^2)$; $|\widehat{w_K}|^2 = max(|h_1|^2, |h_2|^2, \dots, |h_K|^2)$. Based on the derived closed-form formulation we analysed users' different QoS requirements and system throughput.

Now, similar to the equation of P_{OUT}^{K} a CDF of channel gain can be used to calculate the outage probability of instantaneous CSI NOMA DL for user $\widehat{U_\iota}$, where $1 \leq i \leq K$, To do this, we calculate the CDF of the weaker user's channel gain, which equals $\widehat{w_\iota}$ $i \leq j \leq K$. The channel is Rayleigh faded and pdf of which is given as equation no (1), the CDF of $\widehat{U_\iota}$ is

$$F_{|\widehat{W_\iota}|^2}\left(|\widehat{w_\iota}|^2\right) = P\left(\left|\widehat{W_\iota}\right|^2 \leq |\widehat{w_\iota}|^2\right)$$
$$= 1 - P\left(min\{|h_1|^2, |h_2|^2, \dots, |h_K|^2 > |\widehat{w_\iota}|^2\}\right)$$
$$= 1 - \sum_{i=1}^{K} \sum_{k=1}^{i} P\left(\left|\widehat{W_k}\right|^2 > |\widehat{w_\iota}|^2\right)$$

(19)

PDF of which can be given as by incorporating Equation no (1) as

$$F_{|\widehat{W_\iota}|^2}\left(|\widehat{w_\iota}|^2\right) = \sum_{k=1}^{i} \frac{1}{{\delta_k}^2} \sum_{k=1}^{i} e^{-\left(|\widehat{w_\iota}|^2\right)\frac{1}{{\delta_k}^2}}$$

(20)

Similarly, CDF of user $\widehat{U_J}$ is

$$F_{\left|\widehat{W_J}\right|^2}\left(\left|\widehat{w_J}\right|^2\right) = P\left(\left|\widehat{W_J}\right|^2 \leq \left|\widehat{w_J}\right|^2\right)$$
$$= P\left(max\left\{|h_1|^2, |h_2|^2, \dots, |h_K|^2 \leq \left|\widehat{w_J}\right|^2\right\}\right)$$

$$= \sum_{j=1}^{K} \sum_{k=1}^{j} P\left(\left|\widehat{W_k}\right|^2 \leq \left|\widehat{w_J}\right|^2\right)$$

(21)

PDF of which can be given by incorporating Eq. (1) as

$$F_{\left|\widehat{W_J}\right|^2}\left(\left|\widehat{w_J}\right|^2\right) = \sum_{k=1}^{i}\left(1 - e^{-\left(\left|\widehat{w_J}\right|^2\right)\frac{1}{\delta_k{}^2}}\right)$$

(22)

Condition of outage for user $\widehat{U_\iota}$can be written like

$$\widehat{A_\iota} \equiv \begin{array}{c} \left\{log_2\left(1+\frac{\widehat{w_\iota}\widehat{a_J}\widehat{\rho_S}}{1+\widehat{w_\iota}\widehat{\rho_S}\sum_{k=j+1}^{K}\widehat{a_k}}\right)<\widehat{R_\iota}\right\} \\ P\left\{\widehat{w_\iota}<\frac{\widehat{X_\iota}}{\left(\widehat{a_\iota}-\left(\sum_{k=j+1}^{K}\widehat{a_k}\widehat{X_\iota}\right)\widehat{\rho_S}\right)}\right\} \end{array}$$

(23)

Where $\widehat{X_\iota} = 2^{\widehat{R_\iota}} - 1$, which satisfies the constraint$\widehat{X_\iota} < \frac{\widehat{a_J}}{\sum_{k=j+1}^{K}\widehat{a_k}}$, $\widehat{w_\iota}$ and $\widehat{w_J}$ are channel gains of $\widehat{U_\iota}$ and$\widehat{U_J}$, with Rayleigh faded and $E\{|h_i|^2\} = \delta_i{}^2$and CDF of which is given as in Equation (21). $\widehat{A_\iota}$ is the condition of outage probability of $\widehat{U_\iota}$, which is as per definition the outage occurs when the transmission rate exceeds the maximum rate at which the signal $\widehat{U_J}$can successively decoded by $\widehat{U_\iota}$ with interference from other signals i.e. $\widehat{U_{J+1}}, \dots, \widehat{U_K}$, which should be greater than or equal to the desired QoS ($\widehat{R_\iota}\frac{bps}{Hz}$).Similarly, the condition for the outage for the j-th user will be defined as

$$\widehat{A_J} = \begin{cases} log_2\left(1+\frac{\widehat{w_J}\widehat{a_\iota}\widehat{\rho_S}}{1+\widehat{w_J}\widehat{\rho_S}\sum_{k=j+1}^{K}\widehat{a_k}}\right)<\widehat{R_\iota} & j<K \\ log_2\left(1+\widehat{w_J}\widehat{\rho_S}\sum_{k=j+1}^{K}\widehat{a_k}\right)<\widehat{R_J} & j=k \end{cases} \tag{24}$$

In the case of the j-th user, the outage probability is defined as

$$P\left\{\widehat{w_J} < max\left\{\frac{\widehat{X_\iota}}{\left(\widehat{a_\iota}-\widehat{X_J}\sum_{k=j+1}^{K}\widehat{a_k}\right)\rho_S}, \frac{\widehat{X_J}}{\widehat{a_J}\widehat{\rho_S}}\right\}\right\}$$

(25)

Where, $\widehat{X_J} = 2^{\widehat{R_J}} - 1$. The expression for the outage for $\widehat{U_\iota}$ and $\widehat{U_J}$ can be given as

$$\hat{P}_{OUT}^{\widehat{U_\iota}} = \int_0^{\frac{\widehat{X_\iota}}{\left(\widehat{a_\iota}-\left(\sum_{k=j+1}^{K}\widehat{a_k}\widehat{X_\iota}\right)\widehat{\rho_S}\right)}} F_{|\widehat{W_\iota}|^2}(|\widehat{w_\iota}|^2)\ d\left(\left|\widehat{W_\iota}\right|^2\right)$$

(26)

$$\hat{P}_{OUT}^{\widehat{U_J}} = \int_0^{max\left\{\frac{\widehat{X_\iota}}{\left(\widehat{a_\iota}-\widehat{X_J}\sum_{k=j+1}^{K}\widehat{a_k}\right)\rho_S}, \frac{\widehat{X_J}}{\widehat{a_J}\widehat{\rho_S}}\right\}} F_{\left|\widehat{W_J}\right|^2}\left(\left|\widehat{w_J}\right|^2\right) d\left(\left|\widehat{W_J}\right|^2\right)$$

(27)

The Eq. (26) and (27) show individual outage probability analysis for Instantaneous CSI-based ordering.

4. Simulation Results And Discussion

The simulations are conducted to demonstrate the superiority of Instantaneous CSI-based ordering of users in NOMA DL over statistically-based ordering in the form of outage analysis. We present several simulation results for the outage probability analysis of statistical CSI and Instantaneous CSI, along with different QoS requirements of individual users. A Rayleigh faded channel with a path loss component of 2.3 is described with the weakest user at a distance from the base station with the lowest channel gain and the strongest at a distance from the base station with the highest channel gain. The sensitivity factors which we have taken into account are channel gain, power allocation co-efficient, a desired data rate of users, and CSI-based ordering. The other parameters have been described in relevant subsections.

Fig. 1 presents a comparative outage analysis for two users with a statistical and instantaneous CSI-based scheme with fixed power allocation co-efficient as $a_1 = \hat{a}_1 = 0.7$ and $a_2 = \hat{a}_2 = 0.3$, while the targeted data rate for both users is kept as$\tilde{R}_1 = \hat{R}_1 = \tilde{R}_2 = \hat{R}_2 = 1.6\frac{bps}{Hz}$. We can see that Instantaneous CSI outperforms Statistical CSI based on ordered users. There is an exact approximation with the theoretical analysis and the simulated results.

Fig. 2 and 3 represent the relationship between the outage probability and Desired Data-rate. The value of SNR has been kept constant at $18dB$ and $25dB$ respectively. It can be observed that outage performs well with higher SNR values same as also instantaneous CSI gives better outage compared to the statistical CSI. It is also observed that the data-rate value is kept from $0.5\frac{bps}{Hz}$ to $5\frac{bps}{Hz}$, which performs till$3.5\frac{bps}{Hz}$.

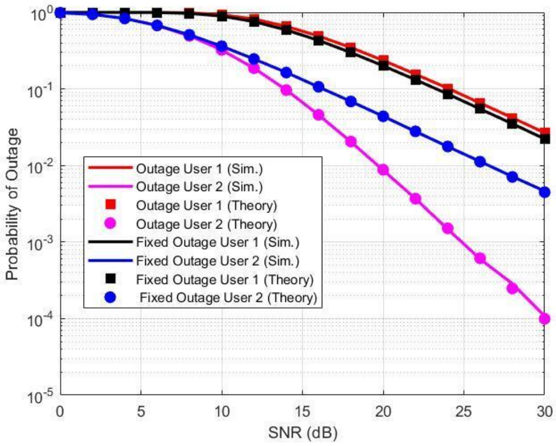

Fig. 1 Statistical and Instantaneous CSI based Outage Analysis for U_1, U_2 and $\widehat{U_1}$, $\widehat{U_2}$

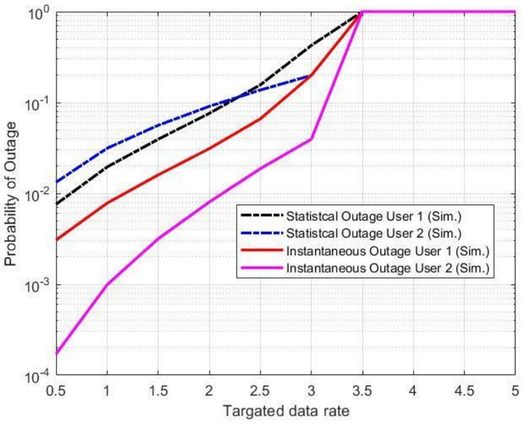

Fig. 2 Outage Analysis vs. Targeted Data-Rate for U_1, U_2 and $\widehat{U_1}$, $\widehat{U_2}$ with Statistical and Instantaneous CSI based Ordering (SNR-18dB).

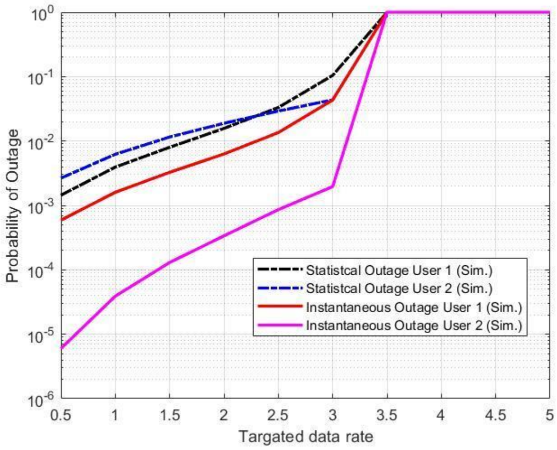

Fig. 3 Outage Analysis vs. Targeted Data-Rate for U_1, U_2 and $\widehat{U_1}$, $\widehat{U_2}$ with Statistical and Instantaneous CSI based ordering (SNR-25dB).

Fig. 4 depicts the relationship between statistical and instantaneous CSI-based outage performance for three users. The allocated fixed power is calculated as $a_1 = \hat{a}_1 = 0.7$, $a_2 = \hat{a}_2 = \frac{2}{3}(1 - a_1)$, and $a_3 = \hat{a}_3 = \left(1 - (a_1 + a_2)\right)$. The channel gains are calculated according to Eq. (1) as $E\{|h_1|^2\} = {\delta_1}^2 = 1$, $E\{|h_2|^2\} = {\delta_2}^2 = 3$, and $E\{|h_3|^2\} = {\delta_3}^2 = 5$. The targeted data rate is kept as $\widetilde{R_1} = \hat{R}_1 = 1\frac{bps}{Hz}$, $\widetilde{R}_2 = \hat{R}_2 = 0.8\frac{\text{bps}}{\text{Hz}}$, and $\widetilde{R}_3 = \hat{R}_3 = 0.7\frac{\text{bps}}{\text{Hz}}$ for U_1, U_2, and U_3 respectively.

We can observe that multiuser interference can be reduced by changing the order and power levels of the individual users and desired QoS can be achieved.

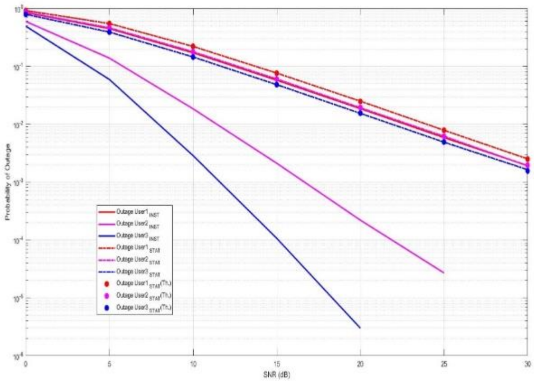

Fig. 4 Statistical and Instantaneous CSI Outage Analysis for U_1,U_2,U_3 and $\widehat{U_1}$,$\widehat{U_2}$,$\widehat{U_3}$.

In Fig. 5 and 6, the relations are shown with varied data-rate requirements for individual users with fixed SNR of $18dB$ and $25dB$ respectively. We can see that the data rate is extending to $1.6\frac{bps}{Hz}$ and the outage seems to be improved at a $25dB$ SNR value.

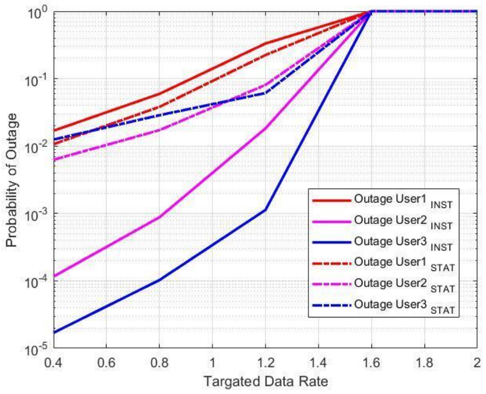

Fig. 5 Outage Analysis vs. Targeted Data-Rate for U_1, U_2 , U_3 and $\widehat{U_1}$, $\widehat{U_2}$, $\widehat{U_3}$ with Statistical and Instantaneous CSI based ordering (SNR-18dB).

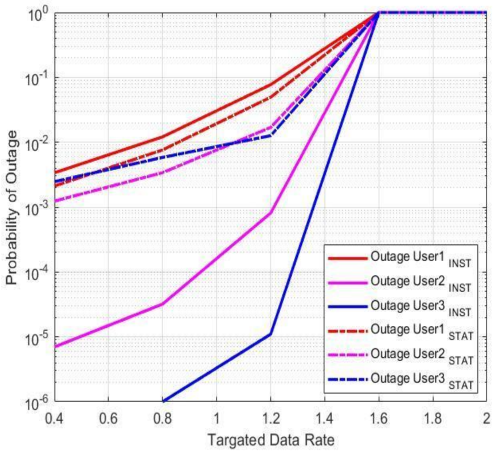

Fig. 6 Outage Analysis vs. Targeted Data-Rate for U_1, U_2 , U_3 and $\widehat{U_1}$, $\widehat{U_2}$, $\widehat{U_3}$ with Statistical and Instantaneous CSI based Ordering (SNR-25dB)

Fig. 7 represents the outage performance over the SNR range for four users, targeted data rates for $U_1, U_2\ , U_3, and\ U_4$ are set as $0.7\ bps/Hz$. The powers are allocated as fixed value as $a_1 = \hat{a}_1 = 0.4$, $a_2 = \hat{a}_2 = 0.3$, $a_3 = \hat{a}_3 = 0.2$ and $a_4 = \hat{a}_4 = 0.1$, with average channel gains as $E\{|h_1|^2\} = {\delta_1}^2 = 1$, $E\{|h_2|^2\} = {\delta_2}^2 = 3$, $E\{|h_3|^2\} = {\delta_3}^2 = 5$ and $E\{|h_4|^2\} = {\delta_4}^2 = 7$.

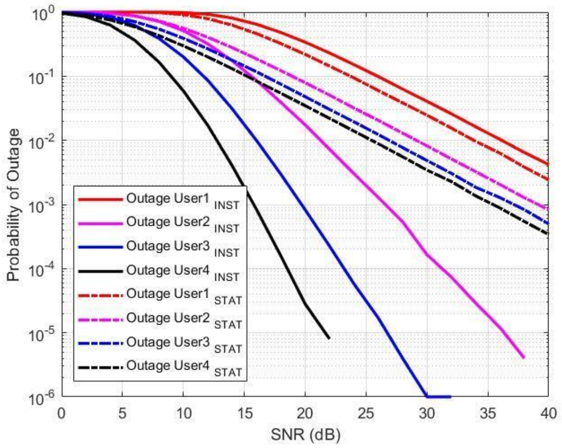

Fig. 7 Statistical and Instantaneous CSI Outage Analysis for U_1, U_2, U_3, U_4 and $\widehat{U_1}$, $\widehat{U_2}$, $\widehat{U_3}$ and $\widehat{U_4}$

Apparently, by increasing the number of users the power is going to be split among users as well as the data rate is affected. It simply depends on the application whether data rate and the number of users are trade-offs.

5. Conclusion

Our study considers both statistical and instantaneous CSI-based user ordering schemes in a Rayleigh faded channel system model with multiple users sharing a single base station to derive an expression for outage probability. For an arbitrary number of users, a single carrier, PDFs and CDFs were formulated for the modelled exponential random variable. Additionally, the averaging of outage probability with the Monte Carlo simulation confirms the accuracy of theoretical expressions. It is clear from the simulation results that an instantaneous CSI-based user ordering scheme can provide better outage performance with desired QoS than a statistical one. Due to different fading distributions between the individual user channels, the users' desired QoS varies greatly based on the application.

References

[1] L. Dai, B. Wang, Y. Yuan, S. Han, C. L. I, and Z. Wang, "Non-orthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74–81, 2015.

[2] S. Li, M. Derakhshani, S. Lambotharan, and L. Hanzo, "Outage Probability Analysis for the Multi-Carrier NOMA Downlink Relying on Statistical CSI," *IEEE Trans. Commun.*, vol. 68, no. 6, pp. 3572–3587, 2020.

[3] D. Do, S. Member, and T. Nguyen, "Joint User Grouping and Decoding Order in Uplink / Downlink MISO / SIMO-NOMA," vol. 8, 2020.

[4] B. Lim, S. S. Nam, Y. Ko, and M. Alouini, "Outage Analysis for Downlink Non-Orthogonal Multiple Access (NOMA) with CDF-Based Scheduling," vol. 2337, no. c, pp. 1–5, 2020.

[5] Z. Tang, J. Wang, J. Wang, and I. J. Song, "On the Achievable Rate Region of NOMA under Outage Probability Constraints," *IEEE Commun. Lett.*, vol. PP, no. c, p. 1, 2018.

[6] J. Wang, B. Xia, K. Xiao, Y. Gao, and S. Ma, "Outage Performance Analysis for Wireless Non-orthogonal Multiple Access Systems," vol. 3536, no. c, pp. 1–8, 2018.

[7] N. Parekh and R. Joshi, "Non Orthogonal Multiple Access Techniques for Next Generation Wireless Networks: A Review," in

Proceedings of the International e-Conference on Intelligent Systems and Signal Processing, 2022, pp. 171–188.

[8] D. Tweed, S. Member, and M. Derakhshani, “Outage-Constrained Resource Allocation in Uplink NOMA for Critical Applications,” vol. 5, no. Dl, 2017.

[9] Z. Yang, Z. Ding, P. Fan, and Z. Ma, “Outage Performance for Dynamic Power Allocation in Hybrid Non-Orthogonal Multiple Access Systems,” *IEEE Commun. Lett.*, vol. 20, no. 8, pp. 1695–1698, 2016.

[10] D. Tse and P. Viswanath, “The wireless channel,” *Fundam. Wirel. Commun.*, pp. 10–48, 2012.

[11] O. Alhussein and P. K. M. Ho, “Performance Analysis of Wireless Fading Channels: A Unified Approach Title: Performance Analysis of Wireless Fading Channels: A Unified Approach,” no. August, 2015.

[12] A. Agarwal, R. Chaurasiya, S. Rai, and A. K. Jagannatham, “Outage Probability Analysis for NOMA Downlink and Uplink Communication Systems with Generalized Fading Channels,” *IEEE Access*, pp. 220461–220481, 2020.