Journal of Harbin Engineering University
ISSN: 1006-7043

Vol 45 No. 1
January 2024

Optimization of A Two-Stage Helical Gearbox with Second Stage Double Gear Sets to Reduce Gearbox Across Section Area and Enhance Gearbox Efficiency

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Abstract
The goal of this research is to look into multi-target optimization of a two-stage helical gearbox with second stage double gear sets (SSDGS) in order to find the most critical design parameters for minimizing gearbox cross section area and enhancing gearbox efficiency. In two phases, the Taguchi technique and grey relation analysis (GRA) were used to address the problem. The single-objective optimization issue was faced first to narrow the separation between variable levels, and then the multi-objective optimization problem was solved to determine the optimal primary design variables. The first and second stage CWFW coefficients, allowable contact stresses (ACS), and first stage gear ratio were also calculated. The study's findings were utilized to identify the best values for five critical design parameters of a two-stage helical gearbox with SSDGS.
Keywords: Helical gearbox, Double gear sets, Optimization, Multi-objective, Gear ratio, Across section area, Gearbox efficiency.

1. Introduction

The gearbox is the most essential part of a mechanical drive system. The objective of this device is to lower the speed and increase the torque sent from the motor shaft to the operating machine shaft. As a result, optimum gearbox design is an ongoing problem.

Until date, the optimal gearbox design has been done on a range of issues. The model developed is utilized in [1] to analyze vibration and noise in the gearbox housing. The authors of [2] conducted research on the possibilities for enhancing energy efficiency and decreasing lifetime costs of electric city buses equipped with multispeed gearboxes. In order to lower the cost of a three-stage helical gear-box, [3] studies the impact of eleven input parameters on second and third stage ratios. [4] outlines a study that optimized partial transmission ratios in mechanical drive systems using a chain and a two-stage helical reducer. [5] runs a simulation to minimize the volume of a two-stage helical gearbox. [6] presented the option of the coefficient of wheel face width for multi-speed helical gear transmissions. [7] also focused on creating hybrid composite gears for a two-stage constant mesh helical gearbox. Recognizing the significance of gearbox cost reduction in both design and construction, [8] computed cost for two-stage helical gearboxes with second-stage double gear-sets utilizing component mass. [9] proposed two methods for determining optimal gear ratios to minimize helical reducer cross-sectional area in the same region of concern. [10] studied the modal characteristic of gearbox housing with applied load. Apart from that, [11] conducted a simulation experiment to study the link between partial gear ratios and input factors, from which models for splitting the total gear ratio of a two-step helical reducer were established. [12] have investigated the use of airborne sound for monitoring the condition of a multi-stage helical gearbox. [13] described a one-of-a-kind study that employed optimization and regression techniques to determine the ideal partial ratios of three-step helical gearboxes with second-step double gear-sets. Furthermore, [14] proposed optimum gear ratios for mechanically driven systems utilizing a two-stage helical gearbox with double gear sets in the first stage and a chain drive to achieve the shortest system length. [15] investigates optimal gear ratios for a two-stage helical gearbox with double gear sets in the first stage.

The purpose of this research is to explore at multi-target optimization learning for a two-stage helical gearbox utilizing SSDGS. In this effort, two single goals were pursued: reducing gearbox across section area and increasing gearbox efficiency. In addition, the CWFW for both stages, the ACS for both stages, and the gear ratio for the first stage were also examined. Furthermore, the multi-objective optimization problem in gearbox design was solved in two stages by combining the Taguchi approach with the GRA. The best values for five critical design parameters were also provided for designing a two-stage helical gearbox with SSDGS.

2. Optimization problem

2.1 Gearbox across section area determination

The across section area Ac of the gearbox is determined by (Fig. 1):

$$A_c = L \cdot H \qquad (1)$$

In which, L, and H can be found by (Fig. 1):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + \frac{d_{w22}}{2} + 2 \cdot S_G + 2 \cdot k \quad (2)$$

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot S_G \qquad (3)$$

In (1), $k = 8 \div 12$ [6]; d_{w11}, d_{w22} are gear pitch diameters of stages 1 and 2 which can be calculated by [6]:

$$d_{w11} = 2\, a_{w1}/(u_1 + 1) \qquad (4)$$

$$d_{w21} = 2 \cdot a_{w1} \cdot u_1/(u_1 + 1) \qquad (5)$$

$$d_{w22} = 2\, a_{w2} u_2\, /(u_2 + 1) \qquad (6)$$

In which, a_{w1} and a_{w2} are the center distances and b_{w1} and b_{w2} are the gear width of stage 1 and 2. These elements is found by [6]:

$$a_{w1} = k_a (u_1 + 1) \sqrt[3]{T_{11} k_{H\beta 1}/([\sigma_{H1}]^2 u_1 X_{ba1})} \qquad (7)$$

$$a_{w2} = k_a (u_2 + 1) \sqrt[3]{T_{12} k_{H\beta 2}/([\sigma_{H2}]^2 u_2 X_{ba2})} \qquad (8)$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \qquad (9)$$

$$b_{w2} = X_{ba2} \cdot a_{w1} \qquad (10)$$

In the above equation, $k_a = 43$ is the material coefficient [6]; $k_{H\beta 1}$ and $k_{H\beta 2}$ are the contacting load ratio for pitting resistance of stages 1 and 2; $k_{H\beta 1} = 1.0 \div 1.06$ and $k_{H\beta 2} = 1.02 \div 1.28$ [6]. $[\sigma_{H1}]$ and $[\sigma_{H2}]$ are ACS (MPa) and u_1 and u_2 are the gear ratios of stages 1 and 2. X_{ba1} and X_{ba2} are CWFW and T_{11} and T_{12} are the pinion torque (Nmm) of stages 1 and 2:

$$T_{11} = T_{out}/\left(u_g \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \qquad (11)$$

$$T_{12} = T_{out}/\left(2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \qquad (12)$$

Where, T_{out} is the output torque (N.mm); η_{hg} is the helical gear efficiency ($\eta_{hg} = 0.96 \div 0.98$); η_b is the rolling bearing efficiency (η_h=0.99÷0.995) [6].

2.2 Gearbox efficiency determination

The efficiency of the gearbox is calculated by:

$$\eta_{gb} = \frac{100 \cdot P_l}{P_{in}} \qquad (13)$$

With P_l is the total gearbox power loss [16]:

$$P_l = P_{lg} + P_{lb} + P_{ls} \qquad (14)$$

In which, P_{lg} is overall gear power loss; P_{lb} is bearing power loss; P_{ls} is seal power loss which are determined as follows:

+) The power loss in gears:

$$P_{lg} = \sum_{i=1}^{2} P_{lgi} \qquad (15)$$

Where, P_{lgi} is the power losses in gears of i stage:

$$P_{lgi} = P_{gi} \cdot \left(1 - \eta_{gi}\right) \qquad (16)$$

In where, η_{gi} is the anticipated gear efficiency of the i stage [17]:

$$\eta_{gi} = 1 - \left(\frac{1+1/u_i}{\beta_{ai}+\beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot (\beta_{ai}^2 + \beta_{ri}^2) \qquad (17)$$

In (16), u_i is the gear ratio of i stage; f is the friction coefficient; β_{ai} and β_{ri} are the arcs of approach and retreat on the i stage which are found by [17]:

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2\right)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}} \qquad (18)$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2\right)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}} \qquad (19)$$

In where, R_{e1i} and R_{e2i} are the outer radiuses of the pinion and gear; R_{1i} and R_{2i} are the pitch radiuses of the pinion and gear; α is the pressure angle; R_{01i} and R_{01i} are the base-circle radiuses of the pinion and gear.

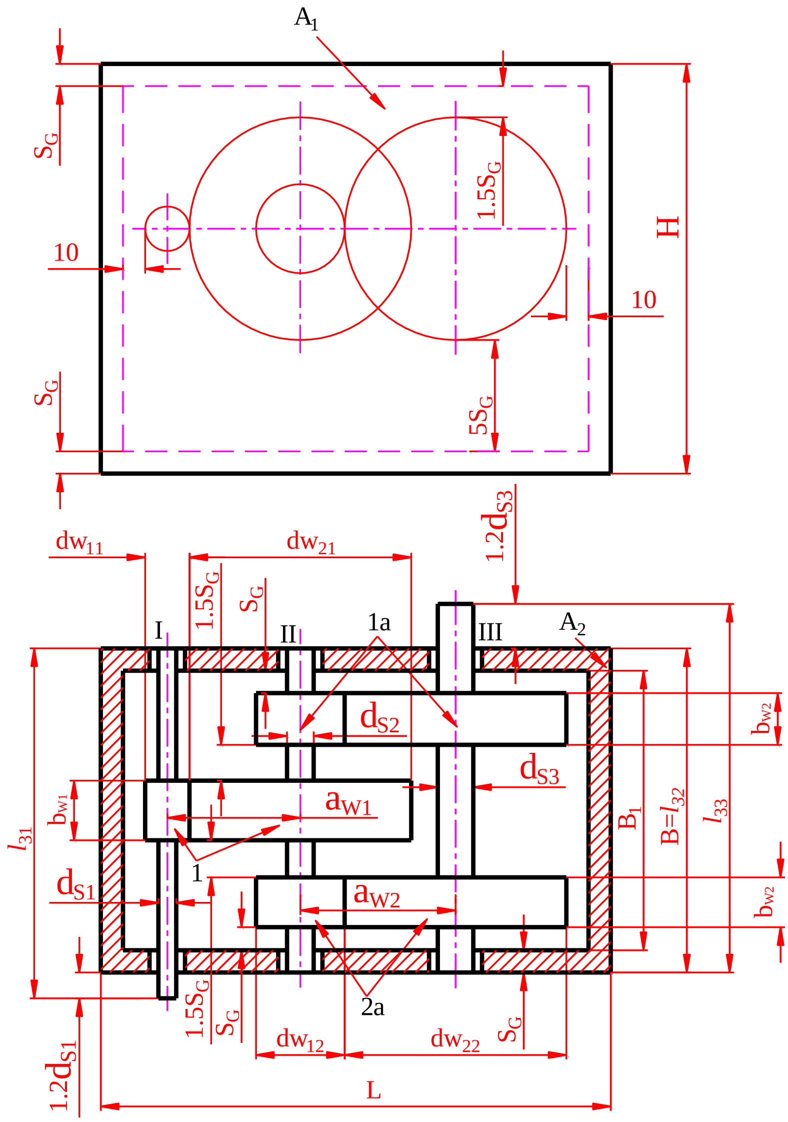

Fig. 1. Calculated schema

The friction coefficient f in Equation (16) is determined by [18]:

- If the sliding velocity v ≤ 0.424 (m/s):

$$f = -0.0877 \cdot v + 0.0525 \qquad (20)$$

- If the sliding velocity v > 0.424 (m/s) :

$$f = 0.0028 \cdot v + 0.0104 \qquad (21)$$

+) The bearing power loss is found by [16]:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \qquad (22)$$

Where, $f_b = 0.0011$ is the bearing friction coefficient (as radical ball bearings with angular contact were selected) [16]; F is the bearing load (N); v is the peripheral speed; i is the bearing ordinal number (i = 1÷6).

+) The total power loss in seals is found by [16]:

$$P_s = \sum_{i=1}^{2} \mathrm{P_{si}} \qquad (23)$$

With P_{si} is the single seal power loss (w):

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\, log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \qquad (24)$$

In which VG_{40} is the ISO Viscosity Grade number.

2.3 Objective function and constrains

2.3.1. Objectives Functions

The multi-objective optimization problem has two following objectives:

Minimizing the gearbox across section area:

$$\min f_2(X) = \mathrm{A}_c \qquad (25)$$

Maximizing the gearbox efficiency:

$$\min f_1(X) = \eta_{gb} \qquad (26)$$

Where X is design variable vector. In this work, five main design parameters including u_1, Xba_1, Xba_2, AS_1, and AS_2 were selected as variables. Therefore, we have:

$$X = \{u_1, Xba_1, Xba_2, AS_1, AS_2\} \quad (27)$$

2.3.2. Constrains

The multi-objective function consists of the following constraints:

$$1 \leq u_1 \leq 9 \text{ and } 1 \leq u_2 \leq 9 \quad (28)$$

$$0.25 \leq k_{be} \leq 0.3 \text{ and } 0.25 \leq X_{ba} \leq 0.4 \quad (29)$$

$$350 \leq AS_1 \leq 420 \text{ and } 350 \leq AS_2 \leq 420 \quad (30)$$

3. Methods

Five major design criteria were chosen for consideration in this study. Table 1 displays the minimum and maximum values for various variables. To solve the optimization problem, the Taguchi technique and grey relation analysis were applied. The L25 (5^5) design was used to optimize the number of levels for each variable. However, among the variables considered, u1 has an almost wide range (u1=1÷9 - Table 1). The difference in the values of these attributes remained substantial even with five levels (in this example, the difference is ((9-1)/4 = 2).

Table 1. Main design factors and their maximum and minimum limits

Factor	Notation	Lower limit	Upper limit
Gear ratio of stage 1	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.3
CWFW of stage 2	X_{ba2}	0.25	0.4
ACS of stage 1 (MPa)	AS_1	350	420
ACS of stage 2 (MPa)	AS_2	350	420

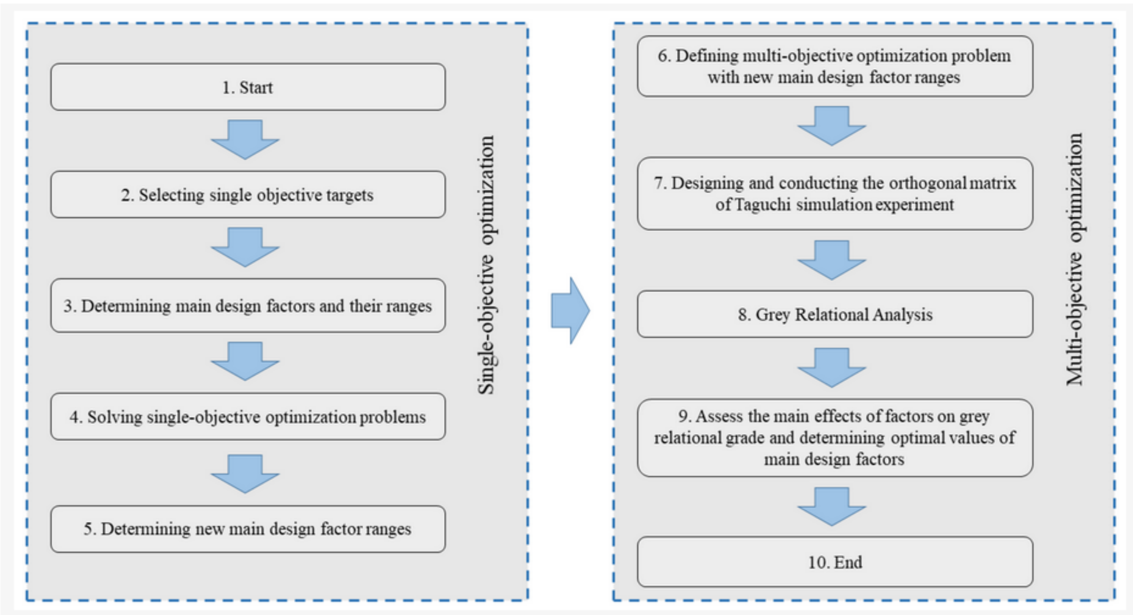

Fig. 2. Method to solve multi-objective problem [18]

In order to reduce the gap between variable values spread throughout a wide range, the 2-stage multi-objective optimization problem solution technique [19] was utilized. The first stage of this method addresses a single-objective optimization problem, while the second addresses a multi-objective optimization problem to discover the optimal core elements of design.

4. Single-objective Optimization

The direct search technique is implemented in this work to address the single-objective op-timization problem. A Matlab-based computer program was also created to address two single-objective problems: reducing gearbox across section area and maximizing gearbox efficiency. Figure 4 depicts a relationship between the optimal gear ratio of the first stage u1 and the overall gearbox ratio ut based on the program's findings. In addition, as shown in Table 2, new restrictions for the variable u1 have been created.

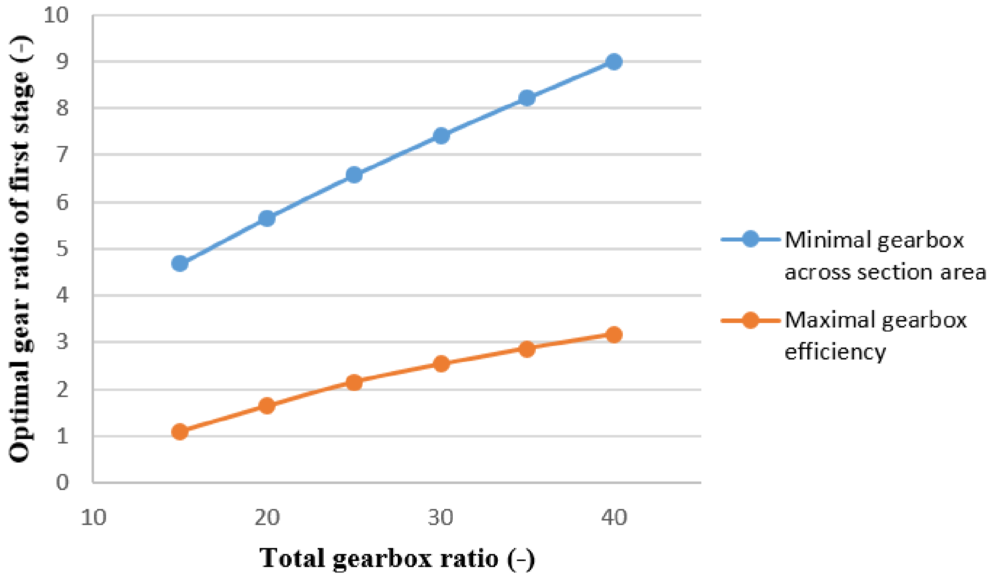

Fig. 3. Optimal gear ratio of first stage versus total gearbox ratio

Table 2. New constraints of u1

u_t	u_1	
	Lower limit	Upper limit
15	1.09	4.68
20	1.63	5.66
25	2.14	6.57
30	2.52	7.42
35	2.86	8.22
40	3.16	9.00

5. Multi-objective Optimization

The purpose of the multi-objective optimization problem for a two-stage helical gearbox with SSDGS in this study is to discover the optimal primary design variables with a given total gear-box ratio that fulfill two single-objective functions: reducing gearbox across section area and optimizing gearbox efficiency. A computer experiment was conducted out to accomplish this. The primary design features and their values are shown in Table 3 for ut = 20. The Taguchi technique and the L25 (5^5) design were used to create the experimental design, and the data was analyzed using Minitab R18 software. Table 4 displays the experimental design and results for ut = 20.

Table 3. Main design factors and their levels for ut = 20

Factor	**Notation**	**Level**				
		1	2	3	4	5
Gear ratio of stage 1	u_1	1.63	2.3675	3.645	4.6525	5.66
CWFW of stage 1	X_{ba1}	0.25	0.2675	0.275	0.2875	0.3
CWFW of stage 2	X_{ba2}	0.25	0.2875	0.325	0.3625	0.4
ACS of stage 1 (MPa)	AS_1	350	368	386	404	420
ACS of stage 2 (MPa)	AS_2	350	368	386	404	420

Table 4. Experimental plan and output results for u_t = 20

Exp. No.	Input Factors					Ac	η_{gb}
	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2	(dm^2)	(%)
1	1.6300	0.2500	0.2500	350	350	26.246	95.572
2	1.6300	0.2625	0.2875	368	368	22.709	95.540
3	1.6300	0.2750	0.3250	386	386	19.913	95.534
4	1.6300	0.2875	0.3625	404	404	17.652	95.500
5	1.6300	0.3000	0.4000	420	420	15.883	95.498
6	2.6375	0.2500	0.2875	386	404	16.373	95.224
7	2.6375	0.2625	0.3250	404	420	14.536	95.217
8	2.6375	0.2750	0.3625	420	350	16.463	95.275
9	2.6375	0.2875	0.4000	350	368	15.263	95.235
10	2.6375	0.3000	0.2500	368	386	18.378	95.251
11	3.6450	0.2500	0.3250	420	368	14.651	95.016
12	3.6450	0.2625	0.3625	350	386	13.641	95.015
13	3.6450	0.2750	0.4000	368	404	12.193	94.997
14	3.6450	0.2875	0.2500	386	420	14.679	95.035
15	3.6450	0.3000	0.2875	404	350	16.306	95.041
16	4.6525	0.2500	0.3625	368	420	11.707	94.756
17	4.6525	0.2625	0.4000	386	350	12.928	94.794
18	4.6525	0.2750	0.2500	404	368	15.373	94.838
19	4.6525	0.2875	0.2875	420	386	13.445	94.839
20	4.6525	0.3000	0.3250	350	404	12.517	94.785
21	5.6600	0.2500	0.4000	404	386	12.364	94.578
22	5.6600	0.2625	0.2500	420	404	13.109	94.614
23	5.6600	0.2750	0.2875	350	420	13.839	94.585
24	5.6600	0.2875	0.3250	368	350	13.649	94.621
25	5.6600	0.3000	0.3625	386	368	12.357	94.607

Table 5. S/N values for every experiment when u_t=20

Exp. No.	Input Factors					Ac		η_{gb}	
	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2		S/N		S/N
1	1.6300	0.2500	0.2500	350	350	26.246	-28.3813	95.572	39.6066
2	1.6300	0.2625	0.2875	368	368	22.709	-27.1240	95.540	39.6037
3	1.6300	0.2750	0.3250	386	386	19.913	-25.9827	95.534	39.6032
4	1.6300	0.2875	0.3625	404	404	17.652	-24.9359	95.500	39.6001
5	1.6300	0.3000	0.4000	420	420	15.883	-24.0187	95.498	39.5999
6	2.6375	0.2500	0.2875	386	404	16.373	-24.2826	95.224	39.5749
7	2.6375	0.2625	0.3250	404	420	14.536	-23.2489	95.217	39.5743
8	2.6375	0.2750	0.3625	420	350	16.463	-24.3302	95.275	39.5796
9	2.6375	0.2875	0.4000	350	368	15.263	-23.6728	95.235	39.5759
10	2.6375	0.3000	0.2500	368	386	18.378	-25.2860	95.25	39.5774

								1	
11	3.6450	0.2500	0.3250	420	368	14.651	-23.3173	95.016	39.5559
12	3.6450	0.2625	0.3625	350	386	13.641	-22.6969	95.015	39.5558
13	3.6450	0.2750	0.4000	368	404	12.193	-21.7222	94.997	39.5542
14	3.6450	0.2875	0.2500	386	420	14.679	-23.3339	95.035	39.5577
15	3.6450	0.3000	0.2875	404	350	16.306	-24.2469	95.041	39.5582
16	4.6525	0.2500	0.3625	368	420	11.707	-21.3689	94.756	39.5321
17	4.6525	0.2625	0.4000	386	350	12.928	-22.2306	94.794	39.5356
18	4.6525	0.2750	0.2500	404	368	15.373	-23.7352	94.838	39.5396
19	4.6525	0.2875	0.2875	420	386	13.445	-22.5712	94.839	39.5397
20	4.6525	0.3000	0.3250	350	404	12.517	-21.9500	94.785	39.5348
21	5.6600	0.2500	0.4000	404	386	12.364	-21.8432	94.578	39.5158
22	5.6600	0.2625	0.2500	420	404	13.109	-22.3514	94.614	39.5191
23	5.6600	0.2750	0.2875	350	420	13.839	-22.8221	94.585	39.5164
24	5.6600	0.2875	0.3250	368	350	13.649	-22.7020	94.621	39.5198
25	5.6600	0.3000	0.3625	386	368	12.357	-21.8383	94.607	39.5185

Table 6. Values of $\Delta_i(k)$ and $\bar{y}_i$

Exp. No	S/N		Zi		Δ_i (k)		Grey relation value y_i		$\bar{y}_i$
			Ac	ngb					
	Ac	η_{gb}	Reference values		Ac	η_{gb}	Ac	η_{gb}	
			1.000	1.000					
1	-28.3813	39.6066	0.0000	1.0000	1.000	0.000	0.333	1.000	0.667
2	-27.1240	39.6037	0.1793	0.9680	0.821	0.032	0.379	0.940	0.659
3	-25.9827	39.6032	0.3420	0.9620	0.658	0.038	0.432	0.929	0.681
4	-24.9359	39.6001	0.4913	0.9279	0.509	0.072	0.496	0.874	0.685
5	-24.0187	39.5999	0.6221	0.9259	0.378	0.074	0.570	0.871	0.720
6	-24.2826	39.5749	0.5845	0.6511	0.416	0.349	0.546	0.589	0.568
7	-23.2489	39.5743	0.7319	0.6441	0.268	0.356	0.651	0.584	0.618
8	-24.3302	39.5796	0.5777	0.7023	0.422	0.298	0.542	0.627	0.584
9	-23.6728	39.5759	0.6715	0.6621	0.329	0.338	0.603	0.597	0.600
10	-25.2860	39.5774	0.4414	0.6782	0.559	0.322	0.472	0.608	0.540
11	-23.3173	39.5559	0.7221	0.4419	0.278	0.558	0.643	0.473	0.558
12	-22.6969	39.5558	0.8106	0.4409	0.189	0.559	0.725	0.472	0.599
13	-21.7222	39.5542	0.9496	0.4228	0.050	0.577	0.908	0.464	0.686
14	-23.3339	39.5577	0.7198	0.4611	0.280	0.539	0.641	0.481	0.561

15	-24.2469	39.5582	0.5896	0.4671	0.410	0.533	0.549	0.484	0.517
16	-21.3689	39.5321	1.0000	0.1798	0.000	0.820	1.000	0.379	0.689
17	-22.2306	39.5356	0.8771	0.2182	0.123	0.782	0.803	0.390	0.596
18	-23.7352	39.5396	0.6626	0.2626	0.337	0.737	0.597	0.404	0.501
19	-22.5712	39.5397	0.8285	0.2636	0.171	0.736	0.745	0.404	0.575
20	-21.9500	39.5348	0.9171	0.2091	0.083	0.791	0.858	0.387	0.623
21	-21.8432	39.5158	0.9324	0.0000	0.068	1.000	0.881	0.333	0.607
22	-22.3514	39.5191	0.8599	0.0364	0.140	0.964	0.781	0.342	0.561
23	-22.8221	39.5164	0.7928	0.0071	0.207	0.993	0.707	0.335	0.521
24	-22.7020	39.5198	0.8099	0.0435	0.190	0.957	0.725	0.343	0.534
25	-21.8383	39.5185	0.9331	0.0293	0.067	0.971	0.882	0.340	0.611

When dealing with multi-objective optimization issues, the Taguchi and GRA techniques are implemented. The primary phases in this technique are as follows:

+) Determine the signal-to-noise ratio (S/N) using the following equations:

The lower the gearbox across section area, the higher the S/N:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} y_i^2) \quad (31)$$

The higher the S/N, the better the gearbox efficient:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} \frac{1}{y_i^2}) \quad (32)$$

Where, yi is the output result; m=1 is the experimental repetition number. That means no repetitions are needed since this is a simulation. Table 5 shows the computed S/N values for the objectives.

The data quantities are different for the two single-objective functions. The data must be normalized, or brought to a compatible scale, to guarantee comparability. To normalize the data, the normalization value Zij, which ranges from 0 to 1, is employed. The following formula is used to calculate this value:

$$Z_i = \frac{SN_i - min(SN_i, = 1,2,..n)}{max(SN_i, j = 1,2,..n) - min(SN_i, = 1,2,..n)} \quad (33)$$

Where n=25 is the experimental number.

+) The grey relational factor is determined by:

$$\mathrm{y_i(k)} = \frac{\Delta_{\min} + \xi.\Delta_{\max}(k)}{\Delta_i(k) + \xi.\Delta_{\max}(k)} \quad (34)$$

In which, i=1,2,...,n; k=2 is the objective number; $\Delta_j(k) = \|Z_0(k) - Z_j(k)\|$ with $Z_0(k)$ and $Z_j(k)$ are the reference and particular comparison sequence; Δ_{min} and Δ_{max} are the minimum and maximum values of i(k); ζ=0.5 is the characteristic coefficient.

+) Finding the coefficient of grey relations by:

$$\overline{y_i} = \frac{1}{k}\sum_{j=0}^{k} y_{ij}\,(k) \quad (35)$$

Where y_{ij} is the grey relation value of the j^th^ output aim of the i^th^ experiment. For each experiment, the calculated grey relation number y_i as well as the average grey relation value $\overline{y_i}$ was shown in Table 6.

To create harmony among the output elements, a higher average grey relation value is advised. As a result, the objective function of a multi-objective problem can be reduced to a single-objective optimization problem, yielding the mean grey relation value.

The findings of an ANOVA test run to analyze the effect of the key design parameters on the average grey relation value $\overline{y_i}$ are shown in Table 7. Table 7 shows that u_1 has the most influence on $\overline{y_i}$ (46.77%), followed by X_{ba2} (28.13%), AS_2 (9.27%), AS_1 (3.76%), and X_{ba1} (2.47%). Using ANOVA analysis, Table 8 shows the order of the effect of the primary design factors on $\overline{y_i}$.

Table 7. Analysis of variance for means

Analysis of Variance for Means

Source	DF	Seq SS	Adj SS	Adj MS	F	P	C (%)
u1	4	0.042159	0.042159	0.010540	4.87	0.077	46.77
Xba1	4	0.002226	0.002226	0.000557	0.26	0.892	2.47
Xba2	4	0.025361	0.025361	0.006340	2.93	0.161	28.13
AS1	4	0.003387	0.003387	0.000847	0.39	0.807	3.76
AS2	4	0.008353	0.008353	0.002088	0.96	0.514	9.27
Residual Error	4	0.008663	0.008663	0.002166			9.61
Total	24	0.090149					

Model Summary

S	R-Sq	R-Sq(adj)
0.0465	90.39%	42.34%

Table 8. Response table for means

Response Table for Means

Level	u1	Xba1	Xba2	AS1	AS2
1	0.6823	0.6177	0.566	0.6018	0.5796
2	0.582	0.6066	0.5678	0.6218	0.5857
3	0.5841	0.5946	0.6025	0.6033	0.6002
4	0.5967	0.5909	0.6337	0.5853	0.6245
5	0.5669	0.6022	0.642	0.5997	0.6218
Delta	0.1154	0.0268	0.076	0.0365	0.0449
Rank	1	5	2	4	3
Average of grey analysis value: 0.602					

+) Determining optimum main design parameters: The best factor set, in theory, would include essential design features with the highest S/N values. As a result, the impact of the major design elements on the S/N ratio (Fig. 4) was estimated. Furthermore, using the Figure 4 chart, the optimal set of multi-objective parameters (corresponding to the red points) may be easily determined. Table 9 shows the appropriate levels and values for the multi-objective function's essential design variables.

+) Evaluating proposal modeling: Figure 5 depicts the Anderson-Darling technique results, which are used to assess the suitability of the proposed model. The data points corresponding to the experimental observations (shown as blue dots in the graph) fall within the 95% standard deviation zone defined by the top and bottom bounds. Furthermore, the p-value of 0.395 is much greater than the level of significance of α = 0.05. These results demonstrate that the empirical model utilized in this study is suitable for evaluation.

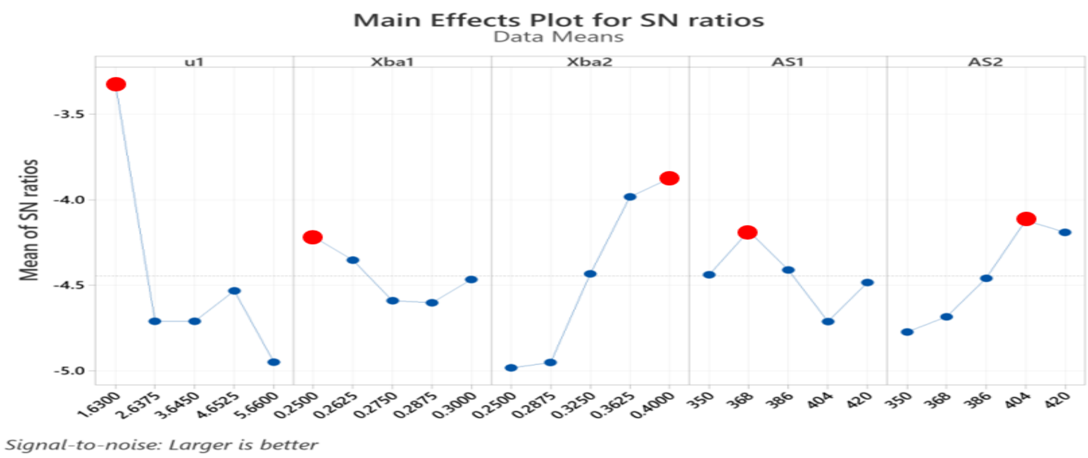

Fig. 4. Main effects plot for S/N ratios

Table 9. Optimum values of main design factors

No.	Input Parameters	Code	Optimum Level	Optimum Value
1	Gear ratio of stage 1	u_1	1	1.63
2	CWFW of stage 1	Xba1	1	0.25
3	CWFW of stage 2	Xba2	5	0.4
4	ACS of stage 1 (MPa)	AS1	2	368
5	ACS of stage 2 (MPa)	AS2	4	404

Table 10. Optimal values of main design factors

No.	**u_t**					
	15	**20**	**25**	**30**	**35**	**40**
u_1	**1.09**	**1.63**	**2.14**	**2.52**	**2.86**	**3.16**
X_{ba1}	**0.25**	**0.25**	**0.25**	**0.25**	**0.25**	**0.25**
X_{ba2}	**0.4**	**0.4**	**0.4**	**0.4**	**0.4**	**0.4**
AS_1	**368**	**368**	**368**	**368**	**368**	**368**
AS_2	**420**	**420**	**420**	**420**	**420**	**420**

Follow in the same way as with ut=20, but with ut values 15, 25, 30, 35, and 40. Table 10 illustrates the optimum values of the five key design parameters at various ut for each of the five primary design parameters. The relation between the ideal first-stage gear ratio and the overall gearbox ratio is depicted in Figure 6. Furthermore, the following regression formula (with R^2=0.999) was presented to obtain the ideal values of u_1:

$$u_1 = 2.1241 \cdot \ln(u_t) - 4.694 \qquad (36)$$

After having u1, the optimal gear ratio of second stage u2 is determined by $u_2=u_t/u_1$.

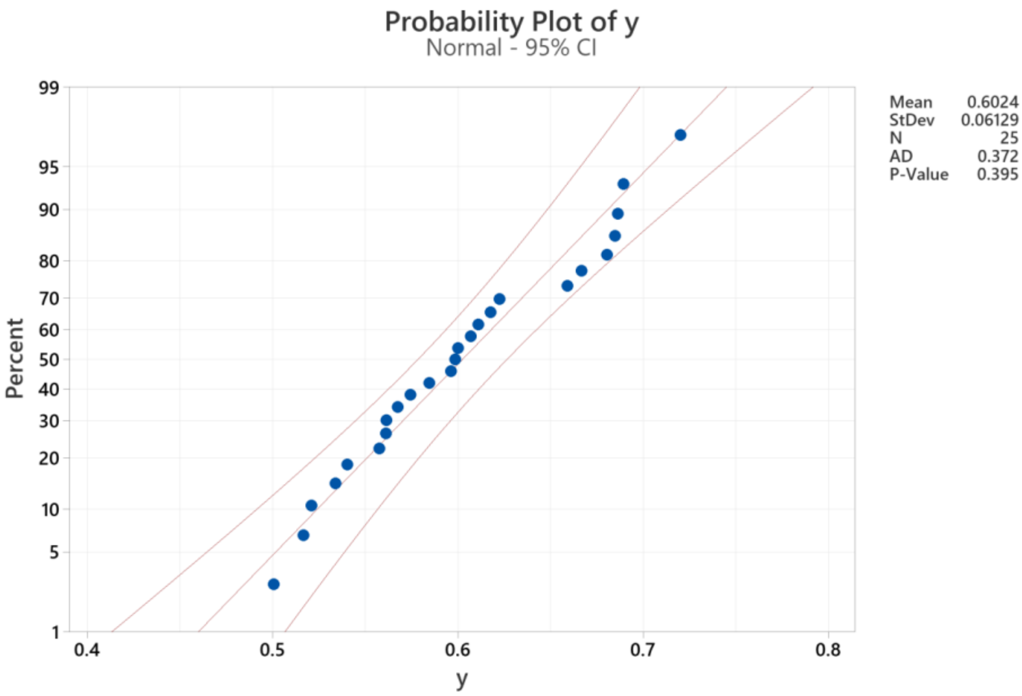

Fig. 5. Probability plot of $\bar{y}$

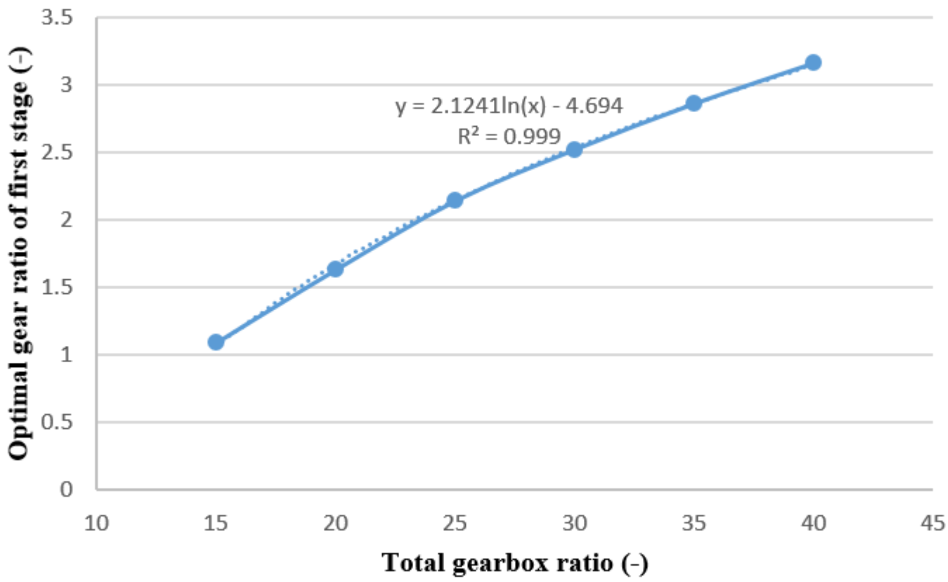

Fig. 6. Optimal gear ratio of first stage versus total gearbox ratio

6. Conclusions

The results of a multi-objective optimization study on optimizing a two-stage helical gearbox with SSDGS to reduce gearbox across section area and improve gearbox efficiency are discussed in this paper. The first stage of this research enhanced the gear ratio, the efficiency of wheel face width in stages 1 and 2, and the permitted contact stress in stages 1 and 2. A simulation experiment based on the Taguchi L25 type was devised and carried out to address this issue. The impact of key design features on the multi-objective goal was also investigated. It was found that u1 has the most influence on $\overline{y_i}$ (46.77%), followed by X_{ba2} (28.13%), AS_2 (9.27%), AS_1 (3.76%), and Xba1 (2.47%). Furthermore, the optimal values for the critical gearbox characteristics have been advised. A regression technique (Equation (36)) was also introduced to discover the optimal first stage u1 gear ratio.

Acknowledgment

This work was supported by Thai Nguyen University of Technology

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