

# Convergence Of An Efficient Iterative Method For Fixed Points Under Mild Assumptions

Mukund Mohan<sup>a</sup>, S. N. Roy<sup>b</sup>

<sup>a,b</sup>University Department of Mathematics, Lalit Narayan Mithila Univeristy, Darbhanga, Bihar, India-846004

email:<sup>a</sup>mukund.m8p@gmail.com(Corresponding Author), <sup>b</sup>snroy14@gmail.com,

**Abstract** Many real-world applications use the concept of mathematical modelling to evaluate them through mathematical equations. Sometimes it uses the fixed point solution. However, the conditions are not so favourable to evaluate at a certain stage. We have made an attempt to use some weaker conditions on a fixed-point iterative method to solve these equations. We have provided the local convergence of such iterative method which we have proposed in [1]. We have used the weaker condition on the derivative and provide the local convergence theorem based on them. This provides the solution when the earlier condition fails. Our results are supported by some numerical examples that we provide here. This justifies the work done by us.

**Keywords:** Striling method; Local convergence; Banach space; Convergence ball; Fixed point.

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## 1. Introduction

Solving nonlinear equations is one of computational mathematics' most significant challenges. For systems that resemble different processes, the difficulty of solving these systems of nonlinear equations emerges in a variety of engineering, mathematics, physics, chemistry, and biological domains. In many instances, the nonlinear issues naturally occur in the form of nonlinear equations or systems of nonlinear equations in Banach spaces. The pursuit of solving nonlinear equations provides ample incentive for scientists to create novel computationally effective iterative strategies. Naturally, the lack of analytical solutions for the majority of nonlinear problem types is a major contributing factor. We go over the creation of iterative strategies for solving scalar nonlinear equations, computing a matrix's generalized inverse, broad classes of nonlinear equation systems, and particular systems of nonlinear equations connected to partial and ordinary differential equations. We've worked with a variety of higher order iterative techniques to solve nonlinear equations.

In order to solve nonlinear equations, it is crucial to design higher order one point and multipoint iterative algorithms that do not require higher order derivatives. Well-known third-order iterative techniques like the Chebyshev, Halley, and Super-Halley methods work with higher-order derivatives that are either nonexistent or difficult to compute.

In this study we are concerned to solve the equation of a fixed point of a nonlinear operator  $G(r)$  that

generally found in the form of

$$r = G(r). \quad (1)$$

Here  $G : \Omega_0 \subset P \rightarrow P$  is a nonlinear map from an open convex domain  $\Omega_0$  to itself. There are other methods which generally used to solve (1) which is given by

$$r_{n+1} = r_n - (I - G'(G(r_n)))^{-1}(r_n - G(r_n)),$$

$$n \geq 0. \quad (2)$$

This method is known as Striling's method. Some excellent works can be found related to (1) and others can be found in some classical and recent studies in [2, 3, 4, 5, 6, 7, 8].

In a recent study [1], we have proposed an algorithm given by

$$s_n = r_n - (I - G'(w_{-1}))^{-1}(r_n - G(r_n)),$$

$$r_{n+1} = s_n - (I - G'(w_n))^{-1}(r_n - G(r_n)),$$

$$n \geq 0. \quad (3)$$

where  $w_{-1} = r_0$  and  $w_n = \frac{r_n + s_n}{2}$ . In this paper, we have shown the order of convergence  $1 + \sqrt{2}$ . Some lemmas and theorems were also presented using Lipschitz and some other continuity conditions.

To make the paper self content, the continuity conditions used in [1] were given by

$$A_*^{-1} = (I - G'(r^*))^{-1}, \quad r^* = G(r^*),$$

$$\| |A_*^{-1}(G'(r) - G'(r^*))| \| \leq J_* \|r - r^*\|,$$

$$\|A_*^{-1}(G'(r) - G'(s))\| \leq J_* \|r - s\| \quad (4)$$

However, there are some problems that comes from the mathematical modelling real life phenomenal which does not satisfies the Lipschitz conditions. For motivational example, let us define a function  $g$  on  $\Omega_0 =$

$$\left[ \frac{-1}{2}, \frac{5}{2} \right] \text{ by } g(r) = \begin{cases} r^3 \ln r^2 + r^5 - r^4, & r \neq 0 \\ 0, & r = 0. \end{cases}$$

Choose  $r^* = 1$ . We have that

$$g'(r) = 3r^2 \ln r^2 + 5r^4 - 4r^3 + 2r^2$$

$$g''(r) = 6r \ln r^2 + 20r^3 - 12r^2 + 10r,$$

$$g'''(r) = 6 \ln r^2 + 60r^2 - 24r + 22,$$

It should be noted that  $g'''(r)$  is unbounded on  $\Omega_0$ . Then the last inequality given above is not satisfied. Hence, the results depending on second derivative can not apply in this case. To remove this obstacle, we have used the  $\omega$ -conditions which is given by

$$\|A_*^{-1}(G'(r) - G'(r^*))\| \leq \omega(\|r - r^*\|).$$

Here  $\omega(\|r - r^*\|)$  is a non-decreasing function on  $R_+$ . Generally, the local convergence is studied on the case  $\omega(\|r - s\|) = J_* \|r - s\|$  and  $\omega(\|r - s\|) = J_* \|r - s\|, \sim q \in (0, 1]$ , denoted as Lipschitz and Hölder continuity conditions, respectively.

Consider another example of an integral equation

$$G(r)(x) = r(x) - 3 \int_0^1 F(u, v) (r(x))^{5/4} + r(x)^2 dv.$$

with  $r(x) \in C[0, 1]$  and  $F(u, v)$  is the Green's function. Then,

$$\|G'(u) - G'(v)\| \leq \frac{15}{32} \|u - v\|^{1/4} + \frac{3}{4} \|u - v\|.$$

It asserts here that this example can not justify Lipschitz or Hölder continuity condition but satisfy the  $\omega$  conditions that be taken as  $\omega(r) = \frac{15}{32} r^{1/4} + \frac{3}{4} r$ . Motivated from these facts, we are presented here to provide the local convergence theorem with the help of  $\omega$ -conditions.

In this paper, We have made an attempt to use some weaker conditions on a fixed point iterative method to solve these equations. We have provided the local convergence of such iterative method which we have proposed in [1]. We have used the weaker condition on the the derivative and provide the local

convergence theorem based on them. This provides the solution when the earlier condition fails.

Finally, we structured the paper as follows: Section 1 provides the introduction and some literature related to this paper. In Section 2, we provide some lemmas and theorem for the convergence of Method 3 under the weaker  $\omega$  condition. Finally some numerical examples are provided in Section 3. Later some conclusions are established of this paper.

## 2 Convergence Analysis

Let us state that we have used  $K(a, b)$  and  $\bar{K}(a, b)$  for open and closed ball with the center at  $a$  and radius  $b$ , respectively. We assume that the following conditions holds true.

$$\begin{aligned} A_*^{-1} &= (I - G'(r^*))^{-1} \sim \text{exists}, \sim r^* = G(r^*), \\ \|A_*^{-1}(G'(r) - G'(s))\| &\leq \omega(\|r - s\|), \\ \|A_*^{-1}(G'(r) - G'(r^*))\| &\leq \omega_*(\|r - r^*\|), r, s, r^* \in \Omega_0 \end{aligned} \quad (5)$$

Here  $\omega$  and  $\omega_*$  are the nondecreasing functions on  $R_+$  with  $\omega(0) = \omega_*(0) = 0$  satisfying  $\omega_*(t) \leq \omega(t)$ .

Now, we define the parameter  $L_0$  by

$$L_0 = \sup(l \geq 0; \quad \omega_*(l) < 1).$$

Let us take some functions as

$$L_1 = \sup(l_1 \geq 0; \quad \omega(l_1) + \omega_*(l_1) < 1),$$

$$L_2 = \sup(l_2 \geq 0; \int_0^1 \omega((1 - 2\theta)l_2) d\theta + \omega_*(l_2) < 1)$$

and

$$L_3 = \sup \left( l_3 \geq 0; \int_0^1 \omega(\|(1 + \theta)l_3\|) d\theta + \omega_*(l_3) < 1 \right).$$

Now, we take element "L" as

$$L = \inf(L_0, L_1, L_2, L_3). \quad (6)$$

Now, we derive some lemmas which will help us to construct the main theorem.

**Lemma 1.** Suppose (5) holds then the following is true

$$\|(I - G'(w))^{-1} A_*\| \leq \frac{1}{1 - \omega_*(\|w - r^*\|)}$$

for  $w, r \in \Omega_0$  and  $\omega_* \in L_0$ .

*Proof:*

$$\begin{aligned} & \|I - A_*^{-1}(I - G'(w))\| = \|A_*^{-1}(I - G'(w) - \\ & (I - G'(r^*)))\| \\ & = \|A_*^{-1}(G'(w) - G'(r^*))\| \leq \omega_*(\|w - r^*\|). \end{aligned}$$

Now, using Banach Lemma [9] on invertible operators, we get

$$\|(I - G'(w))^{-1}A_*\| \leq \frac{1}{1 - \omega_*(\|w - r^*\|)}.$$

Now, the next lemma will establish some inequalities and relationship among elements arises in the consecutive iterations.

**Lemma 2.** Suppose (5) and Lemma 1 hold true then the following relationship are true for the iterates belong to  $\Omega_0$

$$[(a)] \|s_0 - r^*\| \leq \frac{\omega(\|r_0 - r^*\|)}{1 - \omega_*(\|r_0 - r^*\|)} \|r_0 - r^*\| \quad |$$

$$\begin{aligned} [(b)] & \|r_{n+1} - r^*\| \\ & \leq \frac{\int_0^1 \omega \left( \frac{\|(1-2\theta)(r_n - r^*) + s_n - r^*\|}{2} \right) d\theta}{1 - \omega_*(\|w_n - r^*\|)} \|r_n - r^*\| \\ & n = 0, 1, 2, \dots \end{aligned}$$

$$\begin{aligned} [(c)] & \|s_{n+1} - r^*\| \\ & \leq \frac{\int_0^1 \omega \left( \frac{|s_n - r^*| + |r_n - r^*| + 2\theta|r_{n+1} - r^*|}{2} \right) d\theta}{1 - \omega_*(\|w_n - r^*\|)} \|r_{n+1} - r^*\|, \\ & n = 1, 2, \dots \end{aligned}$$

*Proof.* We use the mathematical induction on  $r_n$  to prove this lemma. From (3),

$$\begin{aligned} s_0 - r^* &= r_0 - r^* - (I - G'(r_0))^{-1}(r_0 - G(r_0)) \\ &= (I - G'(r_0))^{-1}((I - G'(r_0))(r_0 - r^*) - (r_0 - G(r_0))) \\ &= (I - G'(r_0))^{-1}(G(r_0) - G(r^*) - G'(r_0)(r_0 - r^*)) \\ &= (I - G'(r_0))^{-1} \int_0^1 (G'(r^* + \theta(r_0 - r^*)) - G'(r_0))(r_0 - r^*) d\theta \\ &= (I - G'(r_0))^{-1} A_* \int_0^1 A_*^{-1}(G'(r^* + \theta(r_0 - r^*)) - G'(r_0))(r_0 - r^*) d\theta. \end{aligned}$$

Taking norm on both sides we get

$$\|s_0 - r^*\|$$

$$\begin{aligned} & \leq \|(I - G'(r_0))^{-1}A_*\| \left\| \int_0^1 A_*^{-1} \left( G'(r^* \right. \right. \\ & \quad \left. \left. + \theta(r_0 - r^*) \right) - G'(r_0) \right) (r_0 - r^*) d\theta \right\| \\ & \leq \frac{\int_0^1 \omega(\|r^* + \theta(r_0 - r^*) - r_0\|) d\theta}{1 - \omega_*(\|r_0 - r^*\|)} \|r_0 - r^*\| \quad (\text{Using Lemma 1}) \\ & = \frac{\int_0^1 \omega(\|(1 - \theta)(r_0 - r^*)\|) d\theta}{1 - \omega_*(\|r_0 - r^*\|)} \|r_0 - r^*\| \\ & < \frac{\omega(\|r_0 - r^*\|)}{1 - \omega_*(\|r_0 - r^*\|)} \|r_0 - r^*\|. \end{aligned}$$

Now,

$$\begin{aligned} r_1 - r^* &= r_0 - r^* - (I - G'(w_0))^{-1}(r_0 - G(r_0) - r^* + G(r^*)) \\ &= (I - G'(w_0))^{-1}(G(r_0) - G(r^*) - G'(w_0)(r_0 - r^*)) \\ &= (I - G'(w_0))^{-1} \left( \int_0^1 (G'(r^* + \theta(r_0 - r^*)) - G'(w_0)) d\theta \right) (r_0 - r^*) \\ &= (I - G'(w_0))^{-1} A_* \left( \int_0^1 A_*^{-1} (G'(r^* + \theta(r_0 - r^*)) - G'(w_0)) d\theta \right) (r_0 - r^*). \end{aligned}$$

Taking norm both sides and by the use of (5) and Lemma 1, we get

$$\begin{aligned} \|r_1 - r^*\| & \leq \frac{\int_0^1 \omega(\|r^* + \theta(r_0 - r^*) - w_0\|) d\theta}{1 - \omega_*(\|w_0 - r^*\|)} \|r_0 - r^*\| \\ & = \frac{\int_0^1 \omega \left( \left\| r^* + \theta(r_0 - r^*) - \frac{r_0 + s_0}{2} \right\| \right) d\theta}{1 - \omega_*(\|w_0 - r^*\|)} \|r_0 - r^*\| \\ & = \frac{\int_0^1 \omega \left( \frac{\|(1 - 2\theta)(r_0 - r^*) + s_0 - r^*\|}{2} \right) d\theta}{1 - \omega_*(\|w_0 - r^*\|)} \|r_0 - r^*\|. \end{aligned}$$

Now,

$$\begin{aligned} s_1 - r^* &= r_1 - r^* - (I - G'(w_0))^{-1}(r_1 - G(r_1) - r^* + G(r^*)) \\ &= (I - G'(w_0))^{-1}(G(r_1) - G(r^*) - G'(w_0)(r_1 - r^*)) \end{aligned}$$

$$\begin{aligned}
 &= (I - G'(w_0))^{-1} \left( \int_0^1 (G'(r^* + \theta(r_1 - r^*)) \right. \\
 &\quad \left. - G'(w_0)) d\theta \right) (r_1 - r^*) \\
 &= (I - G'(w_0))^{-1} A_* \left( \int_0^1 A_*^{-1} (G'(r^* + \theta(r_1 - r^*)) \right. \\
 &\quad \left. - G'(w_0)) d\theta \right) (r_1 - r^*).
 \end{aligned}$$

Taking norm both sides and by the use of (5) and Lemma 1, we get

$$\begin{aligned}
 \|s_1 - r^*\| &\leq \frac{\int_0^1 \omega(\|r^* + \theta(r_1 - r^*) - w_0\|) d\theta}{1 - \omega_*(\|w_0 - r^*\|)} \|r_1 - r^*\| \\
 &= \frac{\int_0^1 \omega(\|r^* + \theta(r_1 - r^*) - \frac{r_0 + s_0}{2}\|) d\theta}{1 - \omega_*(\|w_0 - r^*\|)} \|r_1 - r^*\| \\
 &\leq \frac{\int_0^1 \omega\left(\frac{\|s_0 - r^*\| + \|r_0 - r^*\| + 2\theta\|r_1 - r^*\|}{2}\right) d\theta}{1 - \omega_*(\|w_0 - r^*\|)} \|r_1 - r^*\|.
 \end{aligned}$$

This shows that the lemma is true for "n = 1".

Suppose this is true for k. Now for "k + 1",

$$\begin{aligned}
 r_{k+1} - r^* &= r_k - r^* \\
 &\quad - (I - G'(w_k))^{-1} (r_k - G(r_k) - r^* + G(r^*)) \\
 &= (I - G'(w_k))^{-1} (G(r_k) - G(r^*) - G'(w_k)(r_k - r^*)) \\
 &= (I - G'(w_k))^{-1} \left( \int_0^1 (G'(r^* + \theta(r_k - r^*)) - G'(w_k)) d\theta \right) (r_k - r^*) \\
 &= (I - G'(w_k))^{-1} A_* \left( \int_0^1 A_*^{-1} (G'(r^* + \theta(r_k - r^*)) - G'(w_k)) d\theta \right) (r_k - r^*).
 \end{aligned}$$

Taking norm both sides and by the use of (5) and Lemma 1, we get

$$\begin{aligned}
 \|r_{k+1} - r^*\| &\leq \frac{\int_0^1 \omega(\|r^* + \theta(r_k - r^*) - w_k\|) d\theta}{1 - \omega_*(\|w_k - r^*\|)} \|r_k - r^*\| \\
 &= \frac{\int_0^1 \omega\left(\|r^* + \theta(r_k - r^*) - \frac{r_k + s_k}{2}\| \right) d\theta}{1 - \omega_*(\|w_k - r^*\|)} \|r_k - r^*\| \\
 &= \frac{\int_0^1 \omega\left(\frac{\|(1 - 2\theta)(r_k - r^*) + s_k - r^*\|}{2}\right) d\theta}{1 - \omega_*(\|w_k - r^*\|)} \|r_k - r^*\|.
 \end{aligned}$$

Now,

$$\begin{aligned}
 s_{k+1} - r^* &= r_{k+1} - r^* \\
 &\quad - (I - G'(w_k))^{-1} (r_{k+1} - G(r_{k+1}) - r^* + G(r^*)) \\
 &= (I - G'(w_k))^{-1} (G(r_{k+1}) - G(r^*) - G'(w_k)(r_{k+1} - r^*)) \\
 &= (I - G'(w_k))^{-1} \left( \int_0^1 (G'(r^* + \theta(r_{k+1} - r^*)) - G'(w_k)) d\theta \right) (r_{k+1} - r^*) \\
 &= (I - G'(w_k))^{-1} A_* \left( \int_0^1 A_*^{-1} (G'(r^* + \theta(r_{k+1} - r^*)) - G'(w_k)) d\theta \right) (r_{k+1} - r^*).
 \end{aligned}$$

Taking norm both sides and by the use of (5) and Lemma 1, we get

$$\begin{aligned}
 \|s_{k+1} - r^*\| &\leq \frac{\int_0^1 \omega(\|r^* + \theta(r_{k+1} - r^*) - w_k\|) d\theta}{1 - \omega_*(\|w_k - r^*\|)} \|r_{k+1} - r^*\| \\
 &= \frac{\int_0^1 \omega(\|r^* + \theta(r_{k+1} - r^*) - \frac{r_k + s_k}{2}\|) d\theta}{1 - \omega_*(\|w_k - r^*\|)} \|r_{k+1} - r^*\| \\
 &\leq \frac{\int_0^1 \omega\left(\frac{\|s_k - r^*\| + \|r_k - r^*\| + 2\theta\|r_{k+1} - r^*\|}{2}\right) d\theta}{1 - \omega_*(\|w_k - r^*\|)} \|r_{k+1} - r^*\|.
 \end{aligned}$$

This shows that lemma is true for all n.

Theorem 1. Suppose that the Lemma 1 and Lemma 2 hold true and  $r^* \in \Omega_0$ , such that assumptions (5) are satisfied and  $\bar{\mathfrak{K}}(r^*, L) \subseteq \Omega_0$ , where the radius L is given above. Then starting with  $r_0 \in \mathfrak{K}(r^*, L)$ , the sequence  $\{r_n\}$ ,  $\{s_n\}$  generated from (3) belongs to  $\mathfrak{K}(r^*, L)$  and converges to  $r^*$ .

Proof:- Let us define some functions  $f_1$ ,  $f_2$  and  $f_3$  by

$$f_1(r - r^*) = \frac{\omega(\|r - r^*\|)}{1 - \omega_*(\|r - r^*\|)},$$

$$f_2(r - r^*) = \frac{\int_0^1 \omega \left( \frac{\|(1 - 2\theta)(r - r^*) + r - r^*\|}{2} \right) d\theta}{1 - \omega_*(\|w_k - r^*\|)},$$

and

Examples	L <sub>0</sub>	L <sub>1</sub>	L <sub>2</sub>	L <sub>3</sub>	L(radiusofconvergence)
1	0.5820	0.4140	0.5820	0.1371	0.1371
2	0.5820	0.1839	0.2795	0.1371	0.1371
3	0.4798	0.3938	0.4798	0.2758	0.2758

$$f_3(r - r^*) = \frac{\int_0^1 \omega((1 + \theta)\|r - r^*\|) d\theta}{1 - \omega_*(\|r - r^*\|)}.$$

From the definition of 'L', it is clear that  $f_1(r - r^*) < 1$ . So, from the above Lemma 2

$$\|s_0 - r^*\| < \|r_0 - r^*\|$$

and it follows  $f_2(r - r^*) < 1$  and  $f_3(r - r^*) < 1$  which gives

$$\|r_1 - r^*\| < \|s_0 - r^*\| < \|r_0 - r^*\|,$$

$$\|s_1 - r^*\| < \|r_1 - r^*\| < \|s_0 - r^*\| < \|r_0 - r^*\|.$$

respectively. Now,

$$\|r_{n+1} - r^*\| < \dots < f_1^{n+1} \|r_0 - r^*\|.$$

This implies that  $\|r_{n+1} - r^*\| \rightarrow 0$  as  $n \rightarrow \infty$ . Similarly, one can also show that  $\|s_n - r^*\| \rightarrow 0$  as  $n \rightarrow \infty$ . This proves the theorem.

### 3 Numerical Examples

In this section, we provide some numerical examples to find the theoretical results provided by us which have been taken from [10] and [11].

#### Example 2.

Consider the function  $G$  on  $\Omega_0 = \mathfrak{K}(0,1)$  by

$$G(r) = e^r - r - 1$$

Here  $r^* = 0$ . We have  $\omega(r) = (e - 2)r$  and  $\omega_*(r) = (e - 1)r$ .

#### Example 3.

Consider the function  $G$  on  $\Omega_0 = \mathfrak{K}(0,1)$  by

$$G(r_1, r_2, r_3) = \left( e^{r_1} - 1, \frac{e - 1}{2} r_2^2 + r_2, r_3 \right)^T$$

Here  $r^* = (0,0,0)^T$ . We have  $\omega(r) = (e + 1)r$  and  $\omega_*(r) = (e - 1)r$ .

Back to the example given in the introduction part of this paper, we see that  $r^* = 0$  and  $\|I - G'(r^*)\| = 3/4$ . We have  $\omega(r) = \omega_*(r) = \frac{5}{8}r^{1/4} + r$ . Now, we estimate the values of the parameters

evolving in these examples and the radius of convergence in the

following Table 1.

Table1: Values of parameters and radius of convergence balls as in theTheorem1

### Conclusions:

The idea of mathematical modeling is used in many real-world applications to assess them using mathematical formulas. It utilizes the fixed point solution occasionally. Nevertheless, there are moments when the circumstances are not ideal for evaluation. We have attempted to resolve these equations by applying certain weaker constraints on an iterative fixed-point approach. The local convergence of the iterative approach given in [1] has been supplied. The theorem is based on the usage of the weaker condition on the derivative. When the previous condition is not met, this offers the solution. We present some numerical examples here to corroborate our conclusions. This validates the job that we have done.

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