Journal of Harbin Engineering University
ISSN: 1006-7043

Vol 46 No. 4
April 2025

An Analysis of Non-Instantaneous Deteriorating Items with Credit Policy under Preservation Techniques and No Shortages

K. Sheela[1*] & E. Rathnakumari[2]

[1]Research Scholar (Part time), L.N.Government College (Autonomous), Ponneri, Affiliated to University of Madras, Tamil Nadu, India.

[1]Assistant Professor, Department of Mathematics, Anna Adarsh College for Women (Autonomous), Chennai, Tamil Nadu, India.

[2]Government Arts College, Nandanam (Autonomous), Chennai, Affiliated to University of Madras, Tamil Nadu, India.

Abstract

The Model outlined in this paper aims to provide insights into optimizing the inventory management of non-instantaneous deteriorating items inventory with preservation technique and credit based payment policy. This model does not accommodate shortages. Credit policy refers to the permissible time frame within which payment can be made without incurring penalties or adverse consequences. In business and financial transactions, it is common for parties to agree upon specific terms regarding the timing of payments. In day-to-day life, buyers are provided with a fixed timeframe to settle the payment for received goods. Retailers are not imposed any interest for the goods bought from the suppliers during this credit period. The model assumes a uniform demand rate and a constant deterioration rate of the product. Mathematical formulation is outlined for three major scenarios. The primary objective of this Economic Order quantity (EOQ) model is to obtain maximum total profit of the overall inventory by establishing an optimal replenishment policy. Numerical examples are included to illustrate theoretical concepts.

Keywords: Non-instantaneous deterioration, Preservation technique, Credit policy, Economic order quantity.

1. Introduction

Ford.W.Harris, developed the classic Economic order quantity (EOQ) formula in 1913.The term inventory, refers to raw materials, finished or semi-finished goods / products. Success of a company depends on managing the costs associated with inventory efficiently, irrespective of small scale or large scale business. Customers play a crucial role in any firm. Various strategies viz, discounts, free gifts, are also employed to retain the good will of the customers and cash flow. Here, credit policy is applied to enhance the business. In Inventory, deterioration refers to the decline in the quality or condition of goods held in stock over time. This can happen for various reasons, including exposure to environmental factors, expiration, obsolescence, or simply due to the nature of the products. Different types of inventory may be susceptible to deterioration, such as perishable goods, outdated technology or products sensitive to environmental conditions. Non-instantaneous deterioration in inventory refers to the process by which the quality or condition of goods gradually declines over time rather than experiencing an immediate and abrupt change. Products like vegetables, fruits, processed foods, fall under this category. Proper inventory management and preservation technique helps to reduce the risk of deterioration. Goyal S.K [5] was the first to derive economic order quantity under permissible delay in payment for replenishment, time interval and order quantity. Chandan Mahato and Gour Chandra Mahata [3] determined the optimal replenishment policy with preservation cost using price function and illustrated numerical examples to substantiate the proof under credit policy. Jayashri & Umamaheswari [7] explored EOQ model on perishable items with partial trade credit and preservation technique. Yung-Fu-Huang

[10] proposed the model when a partial permissible payment is granted for a small order quantity. Mahato [6] proposed EOQ for deteriorating items with credit policy and expiration dates. Arash [2] insisted on green technology by exploring carbon emissions of warehouse operations for deteriorating items with credit policy. Morris [4] discussed the need of preservation for mushrooms to prevent from decaying after its harvest. Aditya Shastri [1] discussed preservation technology for three categories of deteriorating products when quantity discount was availed and deduced the model. Subhajit Das et al. [8] derived optimal replenishment with partial trade credit, no shortages and reliability with deterioration rate for exponential function. Sunil Kumar [9] considered demand as a function of selling price. Necessary and sufficient condition for optimal solution was obtained, incorporating trade credit and preservation technology. After extensive study of the papers, research gap has been identified for integrating the assumed parameters. The present paper investigates the model for non-instantaneous deteriorating items with preservation cost and credit policy, when shortages are not allowed; demand is uniform and constant deterioration rate. Section 2 lists assumptions and notations used in the paper. Mathematical formulation is derived in section 3, theoretical proof for the theorems are derived in section 4. Numerical examples are illustrated in section 5. Conclusion of the study is explained in section 6.

2. Assumptions and Notations

2.1 Assumptions

1. Uniform Demand rate is a known constant.
2. Lead time is zero.
3. Shortages are not allowed.
4. One item is taken into study.
5. Preservation technique is included.
6. Deterioration rate is a constant.
7. Infinite replenishment rate.

2.2 Notations

Notations used throughout this paper are listed below:

$Q(t)$	Inventory level at time t, where $0 \leq t \leq$ T
Z	Demand rate
T	Inventory life cycle
Q_0	Order quantity
δ	Deterioration rate
i_c	Inventory carrying cost in $
P_c	Preservation technique cost in $
i_p	Interest paid in $ per unit time
i_e	Interest earned in $ per unit time
c	Unit cost in $ per unit time
s	Selling cost in $ per unit time
p	Purchase cost in $ per unit time

3. Mathematical Formulation

In this scenario, we consider the deterioration model with preservation technology without shortages and dual credit policy for the interval (0, T). Under the credit policy, partial sum of α is paid by the retailer to the supplier at the time of receiving the product and the remaining within his credit period A. Likewise, partial sum of β is paid by the customer to the retailer at the time of receiving the product and the remaining within his credit period B.

Differential equation which gives the condition of Inventory $Q(t)$ in interval (0, T) is

$$\frac{d(Q(t))}{dt} + \delta Q(t) = -Z \tag{3.1}$$

Here, depletion occurs due to demand and deterioration.

General solution of (1) is

$$Q(t) = -\frac{Z}{\delta} + Ce^{-\delta t} \tag{3.2}$$

Applying boundary conditions, $Q(0) = Q_0$, $Q(T) = 0$

Inventory level at time t is

$$Q(t) = \frac{Z}{\delta}\left(e^{\delta(T-t)} - 1\right) \qquad 0 \leq t \leq T \qquad (3.3)$$

Costs associated with the inventory model are as follows

1. Preservation Cost $= P_c$T (3.4)

2. Deterioration cost is

$$d_c = Q_0 - Z = Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) \qquad (3.5)$$

3. Ordering cost $= O_c$ (3.6)

4. Sales revenue

$$S_r = p\int_0^T Zdt = pZT \qquad (3.7)$$

5. Holding Cost

$$H_c = i_c\int_0^T Q_0(t)dt = i_c\frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) \qquad (3.8)$$

6. Interest earned and paid by the retailers depends on the two credit periods A and B. Here, we consider three cases

i) $B \leq A$ and A $\leq T$

ii) $B \leq A$ and A $\leq T$ + B

iii) $B \geq A$ and A $\leq T$

Case 1: $B \leq A$ and A $\leq T$

Interest is gained by the retailers from the following two payments

i) Immediate payment from 0 to A ii) Delayed payment from B to A.

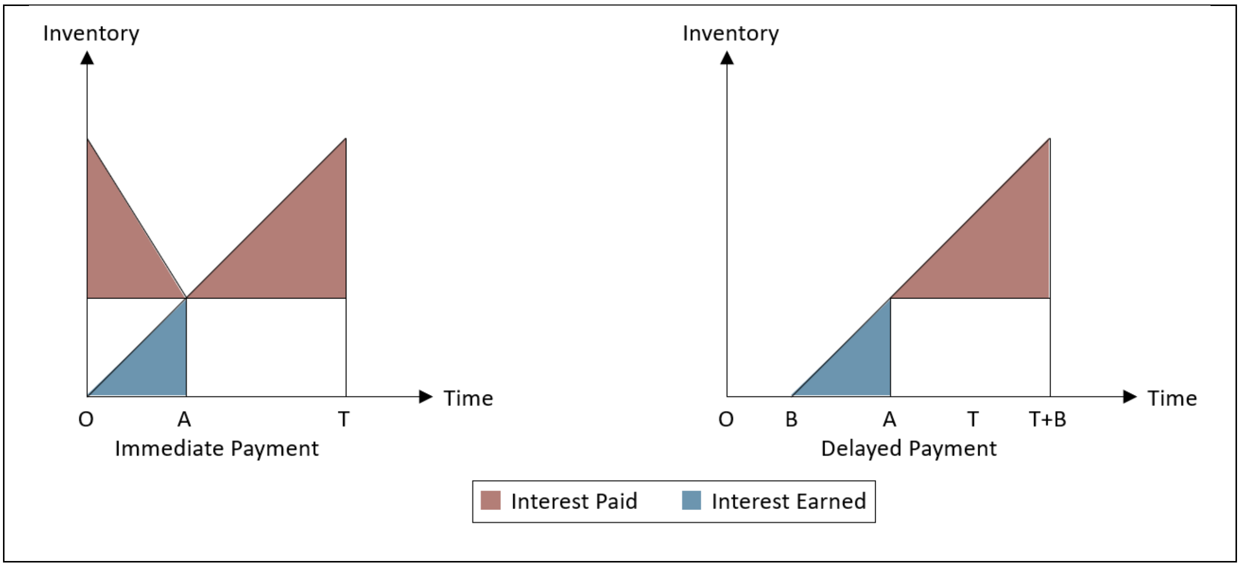

Figure 1: Inventory when $B \leq A$ and $A \leq T$

Interest earned by the retailer= Immediate payment + Delayed payment

$$\text{IE1} = si_e \left[\frac{\alpha Z A^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] \tag{3.9}$$

$$\text{Interest paid IP1} = ci_p \left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2}\right] \tag{3.10}$$

Total Profit

$$\text{TP} = \frac{1}{T}\left[\begin{matrix} sales\ revenue + interest\ earned - (preservation\ cost + \\ deterioration\ cost + ordering\ cost + holding\ cost + interest\ paid)\end{matrix}\right] \tag{3.11}$$

$$TP_1(T) = \frac{1}{T}\left[S_r + IE1 - [P_c + d_c + O_c + H_c + IP1]\right]$$

$$TP_1(T) = \frac{1}{T}\left[pZT + si_e \left[\frac{\alpha Z A^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] - \left[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c \frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p \left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2}\right]\right]\right] \tag{3.12}$$

Case 2 : $B \leq A$ and A $\leq T$ + B

Interest is gained by the retailers from the following two payments

i) Immediate payment from 0 to A ii) Delayed payment from B to A.

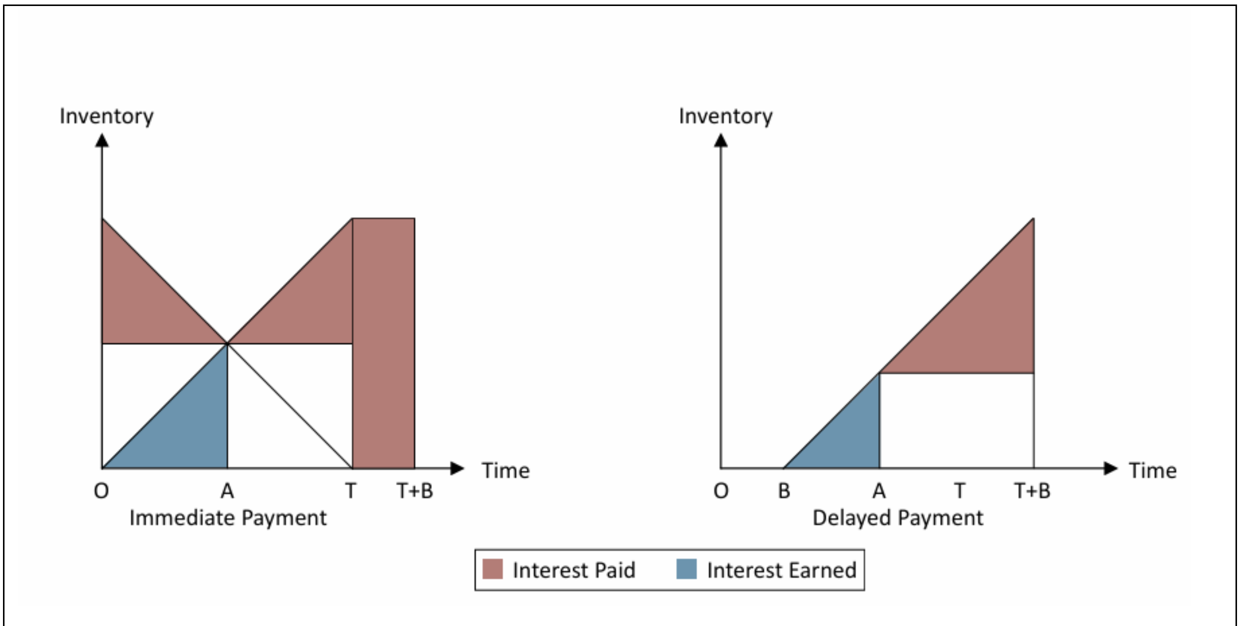

Figure 2: Inventory when $B \leq A$ and $A \leq T + B$

$$\text{IE2} = si_e \left[\frac{\alpha Z A^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] \tag{3.13}$$

$$\text{IP2} = ci_p \left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2} + (1-\beta)ZT(T+B)\right] \tag{3.14}$$

Total Profit

$TP_2(T) = \frac{1}{T}[S_r + IE2 - [P_c + O_c + + H_c + d_c + IP2]]$

$TP_2(T) = \frac{1}{T}\Big[pZT + si_e \left[\frac{\alpha Z A^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] - \Big[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c \frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p \left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2} + (1-\beta)ZT(T+B)\right]\Big]\Big]$
(3.15)

Case 3 : $B \geq A$ **and** A $\leq T$

Interest is gained by the retailers only from immediate payment in the period 0 to A. After the period A, retailers have to finance the amount made by the suppliers in immediate payment and (T + B -A) in delayed payment.

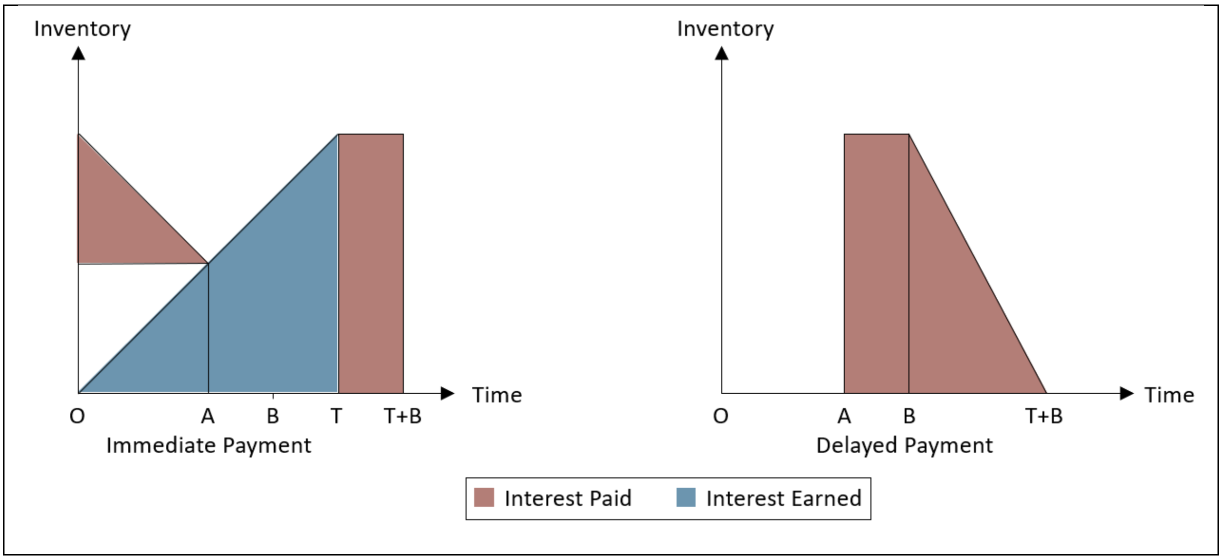

Figure 3: Inventory when $B \geq A$ and $A \leq T$

IE3$= si_e \left[\frac{\alpha Z A^2}{2}\right]$ (3.16)

IP3 $= ci_p \left[\frac{(1-\alpha)ZT^2}{2} + (1-\alpha)ZT(B-A) + (1-\beta)ZT(T+B) + \frac{(1-\beta)ZT^2}{2}\right]$ (3.17)

Total Profit

$TP_3(T) = \frac{1}{T}[S_r + IE3 - [P_c + O_c + + H_c + d_c + IP3]]$

$TP_3(T) = \frac{1}{T}\Big[pZT + si_e \left[\frac{\alpha Z A^2}{2}\right] - \Big[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c \frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p \left[\frac{(1-\alpha)ZT^2}{2} + (1-\alpha)ZT(B-A) + (1-\beta)ZT(T+B) + \frac{(1-\beta)ZT^2}{2}\right]\Big]\Big]$ (3.18)

4. Theoretical Proof

In this section, theoretical proof for obtaining optimal solution of total profit and replenishment cycle for the above three cases are derived. Total profit functions are proved to be concave and Cambini Martein theorem (2009) guarantees that they are maximum and optimal.

Theorem 4.1

If A $\leq T$, there exist a unique maximum solution T_1^* for $TP_1(T)$ and $TP_1(T)$ is concave**.**

Proof :

Consider $g_1 = (TP_1(T))\,T$

$$g_1 = pZT + si_e \left[\frac{\alpha Z A^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] - \left[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c \frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p \left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2}\right]\right] \quad (4.1)$$

Differentiating above equation with respect to T

$$\frac{dg_1}{dT} = pZ - \left[\begin{array}{c} i_c \frac{Z}{\delta^2}\left(\frac{e^{\delta T}}{\delta} - \delta\right) + Z\left(\frac{e^{\delta T}}{\delta^2}\right) + \\ ci_p\left[\alpha Z(T-A) + (1-\beta)ZT + (1-\alpha)Z(T+B-A)\right] \end{array}\right] \quad (4.2)$$

$$\frac{d^2 g_1}{dT^2} = -Z\left[i_c \frac{e^{\delta T}}{\delta^4} + \frac{e^{\delta T}}{\delta^3} + ci_p\,[2-\beta]\right] < 0 \quad (4.3)$$

Hence g_1 is maximum at T_1^*.

From Cambini and Martein (2009) , $TP_1(T)$ is concave.

Value of T_1^* is obtained by equating $\frac{dTP_1(T)}{dT} = 0$

Differentiating ((3.12) with respect to T , we have

$$\frac{dTP_1(T)}{dT} = -\frac{1}{T^2}\left[pZT + si_e \left[\frac{\alpha Z A^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] - \left[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c \frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p \left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2}\right]\right]\right] + pZ - \left[\begin{array}{c} i_c \frac{Z}{\delta^2}\left(\frac{e^{\delta T}}{\delta} - \delta\right) + Z\left(\frac{e^{\delta T}}{\delta^2}\right) + \\ ci_p\left[\alpha Z(T-A) + (1-\beta)ZT + (1-\alpha)Z(T+B-A)\right] \end{array}\right] = 0 \quad (4.4)$$

Hence there exists a unique and maximum T_1^* for $TP_1(T)$.

Theorem 4.2

If A $\leq T$ + B, there exist a unique maximum solution T_2^* for $TP_2(T)$ and $TP_2(T)$ is concave.

Proof:

Consider $g_2 = (TP_2(T))\,T$

$$g_2 = pZT + si_e \left[\frac{\alpha Z A^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] - \left[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c \frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p \left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2} + (1-\beta)ZT(T+B)\right]\right] \quad (4.5)$$

Differentiating above equation with respect to T

$$\frac{dg_2}{dT} = pZ - \left[\begin{array}{c} i_c \frac{Z}{\delta^2}\left(\frac{e^{\delta T}}{\delta} - \delta\right) + Z\left(\frac{e^{\delta T}}{\delta^2}\right) + \\ ci_p\left[\alpha Z(T-A) + (1-\alpha)Z(T+B-A)\right] + (1-\beta)Z(3T+B) \end{array}\right] \quad (4.6)$$

$$\frac{d^2 g_2}{dT^2} = -Z\left[i_c \frac{e^{\delta T}}{\delta^4} + \frac{e^{\delta T}}{\delta^3} + ci_p\,(4-3\beta)\right] < 0 \qquad (4.7)$$

Hence g_2 is maximum at T_2^* and $TP_2(T)$ is concave.

$$\frac{dTP_2(T)}{dT} = -\frac{1}{T^2}\Big[pZT + si_e\left[\frac{\alpha ZA^2}{2} + \frac{(1-\alpha)Z(A-B)^2}{2}\right] - \Big[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c\frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p\left[\frac{\alpha Z(T-A)^2}{2} + \frac{(1-\beta)ZT^2}{2} + \frac{(1-\alpha)Z(T+B-A)^2}{2} + (1-\beta)ZT(T+B)\right]\Big]\Big] + pZ - \left[\begin{array}{c} i_c\frac{Z}{\delta^2}\left(\frac{e^{\delta T}}{\delta} - \delta\right) + Z\left(\frac{e^{\delta T}}{\delta^2}\right) + \\ ci_p\left[\alpha Z(T-A) + (1-\alpha)Z(T+B-A)\right] + (1-\beta)Z(3T+B)\end{array}\right] = 0$$

(4.8)

Hence there exists a unique and maximum T_2^* for $TP_2(T)$.

Theorem 4.3

If A $\leq T$, there exist a unique maximum solution T_3^* for $TP_3(T)$ and $TP_3(T)$ is concave.

Proof:

Consider $g_3 = (TP_3(T))\,T$

$$= pZ + si_e\left[\frac{\alpha ZA^2}{2}\right] - \left[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c\frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p\left[\frac{(1-\alpha)ZT^2}{2} + (1-\alpha)ZT(B-A) + (1-\beta)ZT(T+B) + \frac{(1-\beta)ZT^2}{2}\right]\right] \qquad (4.9)$$

Differentiating above equation with respect to T

$$\frac{dg_3}{dT} = pZ - \left[\begin{array}{c} i_c\frac{Z}{\delta^2}\left(\frac{e^{\delta T}}{\delta} - \delta\right) + Z\left(\frac{e^{\delta T}}{\delta^2}\right) + \\ ci_p\left[(1-\alpha)Z(T+B-A) + (1-\beta)Z(3T+B)\right]\end{array}\right] \qquad (4.10)$$

$$\frac{d^2 g_3}{dT^2} = -Z\left[i_c\frac{e^{\delta T}}{\delta^4} + \left(\frac{e^{\delta T}}{\delta^3}\right) + ci_p\left[4-\alpha-3\beta\right]\right] < 0 \qquad (4.11)$$

Hence g_1 is maximum at T_2^* and $TP_2(T)$ is concave .

$$\frac{dTP_3(T)}{dT} = -\frac{1}{T^2}\left[pZT + si_e\left[\frac{\alpha ZA^2}{2}\right] - \left[P_c + Z\left(\frac{e^{\delta T}}{\delta} - \frac{1}{\delta} - 1\right) + O_c + i_c\frac{Z}{\delta^2}\left(e^{\delta T} - \delta T - 1\right) + ci_p\left[\frac{(1-\alpha)ZT^2}{2} + (1-\alpha)ZT(B-A) + (1-\beta)ZT(T+B) + \frac{(1-\beta)ZT^2}{2}\right]\right]\right] + pZ - \left[\begin{array}{c} i_c\frac{Z}{\delta^2}\left(\frac{e^{\delta T}}{\delta} - \delta\right) + Z\left(\frac{e^{\delta T}}{\delta^2}\right) + \\ ci_p\left[(1-\alpha)Z(T+B-A) + (1-\beta)Z(3T+B)\right]\end{array}\right] = 0 \qquad (4.12)$$

Hence there exists a unique and maximum T_3^* for $TP_3(T)$.

5. Numerical Examples

To illustrate the model described, let us consider the following examples.

Example 5. 1: For Case 1 - Consider A = 0.25 (= 90/365) years, B = 0.1369 (= 50/365) years. Let us consider the following values: $Z = 1500$units/year $P_c =$ 0.02. $i_e = 10\%$ / \$/year,,$i_p = 15$ % / \$/year , $O_c =$ \$20 /order

, $\delta = 0.05$, $i_c = \$2$ / unit/year , p = \$15/unit , c = \$10/unit. Optimal replenishment cycle is $T_1^* = 0.29$,Total profit is $TP_1(T) = \$15377.7$, optimal ordering quantity Q = 116.2

Example 5.2: For Case 2 **–** Consider the same values as in example 1 except A=0.4, B = 0.2 $T_2^* = 0.289$, $TP_2(T) = \$15026.7$, Q = 113.3

Example 5.3: For Case 3 - Consider the same values as in example 1 except A=0.2, B = 0.6

$T_3^* = 0.289$, $TP_3(T) = \$14791.8$, Q = 113.6

Special case: If A = B = 0, then there is no credit policy. In this case, optimal total profit is \$15263.4.

6. Conclusion

The analysis highlights the significant impact of trade credit policy and preservation on the retailer's profit. Analytical expressions for the total profit were derived and optimum replenishment cycle was obtained for various scenarios, involving uniform demand, preservation and credit policy. Numerical example revealed that case 1 yielded the maximum profit for the retailer. The model can be extended to allow shortages. To maintain customer goodwill, these shortages may be either fully or partially backlogged. In addition to time, preservation cost can also be considered as decision variables.

References

[1] Aditya Shastri, Singh S.R, Dharmendra Yadav and Shalley Gupta, "Supply chain management for two-level trade credit financing with selling price dependent demand under the effect of preservation technology", International Journal of procurement Management, Vol.7, No.6, 2014.

[2] Arash Sepehri, "Controllable carbon emissions in an inventory model for perishable items under trade credit policy for credit risk customers", Elsevier, 2021.

[3] Chandan Mahato and Gour Chandra Mahata, "Optimal Replenishment, pricing and preservation technology investment policies for non-instantaneous deteriorating items under two-level trade credit policy", Journal of Industrial and Management Optimization, doi:10.3934/jimo.2021123, 2020.

[4] Dennis P.Murr and Leonard L.Morris, "Effect of storage temperature on post harvest changes in Mushroom", Journal of the American Society for Horticultural science, 100(1), pp. 6-19, 1975.

[5] Goyal S.K, "Economic order quantity under conditions of permissible delay in payments", The Journal of the Operational Research Society, Vol.36 (No.4) pp.335-338, April 1985.

[6] Gour Chandra Mahata, "Optimal ordering policy with trade credit and variable deterioration for fixed lifetime products", International Journal of Operational Research, Jan 2016.

[7] Jayashri.P, Umamaheswari.S, "EOQ inventory model for perishable items with price dependent demand and preservation technique under upstream and downstream trade credit", SCRS Conference proceedings on Intelligent systems pp. 395-413, 2021.

[8] Subhajit Das et.al "A production inventory model with partial trade credit policy and reliability", Alexandria Engineering journal, Elsevier, 60, pp.1325 – 1338, 2021.

[9] Sunil Kumar, Nidhi Handa, S.R. Singh & Dharmendra Yadav "Production inventory model for two-level trade credit financing under the effect of preservation technology and learning in supply chain", Cogent Engineering, 2:1, 1045221, 2015.

[10] Yung-Fu Huang "Economic order quantity under conditionally permissible delay in payments", European Journal of Operational Research, 176, 911-924, 2007.