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Optimization Of Multi-Objective Transportation Problem Using Fuzzy Programming Approach

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Abstract

This study proposes a way for transforming a fuzzy multi-objective transportation problem (FMOTP) into crisp multi-objective transportation problem. The Zimmermann technique has been used to fine the solution for crisp multi-objective transportation problem (CMOTP). This study contrasts outcomes of employing hyperbolic and pentagonal membership functions with those of utilising exponential membership functions. The study demonstrates how real-world issues with inaccurate parameters can be modelled using fuzzy numbers. The study shows how to solve transport problems with competing aims by turning fuzzy problems into deterministic ones.

Keywords: Fuzzy multi-objective LPP; Fuzzy programming technique; Pentagonal fuzzy number; Ranking function; Exponential membership function.

1. Introduction

This study addresses the complex challenges associated with multi-objective optimization in transportation planning in a fuzzy environment. The algorithm ensures that the decision-maker always makes a more informed choice by comparing fuzzy values in an issue using a generic ranking index [11]. In today's dynamic and uncertain business landscape, decision-makers often encounter situations where conflicting objectives must be balanced to achieve optimal outcomes. This research introduces a novel approach that incorporates fuzzy logic to model the inherent uncertainty and imprecision associated with real-world problems. This work extends the current understanding of multi-objective optimization in fuzzy environments, A significant number of real-world issues are intrinsically defined by diverse, contradictory, and incoherent aspects of assessment. Typically, objective functions are used to operationalise these areas of evolution and then optimised within the context of multiple objective linear programming models. Often, the parameters used to solve real-world situations are imprecise numerical values. These scenarios can be well modelled with pentagonal fuzzy quantities. Bellmann and Zadeh are credited with introducing the concepts of fuzzy quantities and fuzzy decision making.

Converting fuzzy linear programming problems into equivalent predictable linear programs is the most popular method for solving them. Modern existence would be impossible without transportation. Mardanya & Roy (2023) investigate the research of Multi-Objective Multi-Item Solid Transportation Problem under fuzzy system. The multi-objective transportation problem (MOTP) can transform into a classical MOTP using order relations, deterministic constraints, and fuzzy programming for solving [5]. Multi-objective periodic routing problem for cash transportation, enhancing security through unpredictable paths and variable arrival times. A new evolutionary algorithm with fuzzy logic and caching is proposed, improving solution quality across objectives like completion times, robbery risk, and customer [19].

In this study author developed a procedure for converting the fuzzy numbers in checklist. Understanding the influence of real-world scenarios, the investigate MMSTP in this instance with parameters treated as trapezoidal fuzzy numbers, such as transportation cost, supply, and demand. The conversation rule is then used to transform trapezoidal fuzzy numbers into approximately approximation interval numbers. An individual cannot avoid transportation-related issues in their daily activity. It is impossible for someone to generate what they need at home to meet everyday demands. Industries, huge farms, etc. can produce items more effectively, but to meet demand, both people and goods must move. In the needs of large-stage objective increases, many authors solved multi-objective problems in different ways to find the solution of the optimal value. One kind of linear programming problem (LPP) is the transportation problem. These days, the decision-maker manages multiple objectives concurrently in the actual world. The concept behind this new model for the weighted goal programming approach to minimise the distances between practical objective space and ideal objectives. It offers Multi Objective Linear Programming Problems (MOLPP) the best compromised answer.

To address MOLPP, the suggested model solves a different single type of objective subproblems in which objectives are converted to constraints [12]. In LPP, fuzzy concepts are utilised to manage data ambiguity and uncertainty. A Modified method for solving kind of fuzzy transportation problem (FTP) using generalised trapezoidal fuzzy numbers (GTpFN) was presented. In that type of problem, the requirements and availability are real numbers, but the decider is unsure of the precise value of transportation charge. The fuzzy multi-objective transportation problem (FMOTP) [1,16,18] proposed algorithm is first transformed into crisp MOTP by the proposed ranking function. Next, the crisp MOTP is changed into single type objective transportation problem utilising the values of the sum of an objective function. Multi objective fuzzy-based problems solved by the Zimmerman technique using exponential membership functions, trapezoidal and hyperbolic membership functions [3]. The multicriteria transportation problem has been the subject of a straightforward mathematical model that uses an exponential membership function rather than a linear one. The model prioritises finding the best possible compromise [5]. Adding a hyperbolic membership function to Zimmermann's fuzzy programming method for multi-objective nonlinear programming problems. Applying a ranking function to transform fuzzy multi-objective nonlinear problems into crisp ones, and linearising nonlinear problems in Zimmermann's method to reduce their complexity [17]. Some different problems of multi-objective are solved by some authors using capacitated transportation [2,13] in which used the fuzzy programming techniques to find optimal solution. Some cases of multi objectivate deterministic multi-objective model with left, centre, and right interval functions replaces the uncertain multi-objective optimisation model. Using intuitionistic fuzzy programming, a conflicting set of objectives is reconciled by considering both the linear and nonlinear degree of membership and non-membership functions [10]. A fuzzy multi-objective optimisation technique utilised to address a multi-objective nonlinear programming issue within the framework of structural design. In a fuzzy context [7,15] created multi-objective structural problem for a planar truss based structural model.

2. TRANSPORTATION PROBLEM WITH SEVERAL OBJECTIVES

2.1 Conceptual model in mathematics

Conceptual model in mathematics. Transporting a homogeneous product from each of m suppliers to n destinations is goal of a classical transportation problem. The sources are supplying points, warehouses, or production facilities, and they are identified by their available capacities $a_u(u = 1,2,\dots,m)$. The locations, which have required demand levels $b_v(v = 1,2,\ \dots,n)$ are warehouses, demand points, or consuming facilities. An infringement the movement of a product unit from source u to destination v is referred to as C_{uv}. The punishment may be in the form of underutilised capacity, delivery time, quantity of items provided, or transportation costs. From origin O_u to destination D_v, an unknown quantity to be conveyed is represented by a variable X_{uv}. Though, transport problems might have more than one goal; they are not always of the single aim variety. Classical linear programming works with discrete parameters. However, the knowledge that is available in the real world is imprecise, unclear, and vague. Fuzzy Sets are used to handle the impreciseness and uncertainty elements and produce ideal answers. Flexible aspiration levels or goals are effectively handled by MOLP. Through fuzzy constraints, FMOLP improves efficacy of solutions under workable solutions. The technique of FMOLP utilised to solve TSP with imprecise & ambiguous parameters [3,11,19]. A mathematical formulation of a multi-objective transportation problem is as follows:

Minimize

$$Z_u = \begin{cases} \sum_{u=1}^{n} \sum_{v=1}^{n} c_{uv}^{1} x_{uv} \\ \sum_{u=1}^{n} \sum_{v=1}^{n} c_{uv}^{2} x_{uv} \\ \\ \sum_{u=1}^{n} \sum_{v=1}^{n} c_{uv}^{p} x_{uv} \end{cases} \quad (2.1)$$

Subject to:

$$\sum_{v=1}^{n} x_{uv} = a_u, \quad u \in \{1,2,\dots,m\} \quad (2.2)$$

$$\sum_{u=1}^{m} x_{uv} = b_v, \quad v \in \{1,2,\dots,n\} \quad (2.3)$$

$$x_{uv} \geq 0 \quad \forall\, u, v \quad (2.4)$$

Z_p refers to a penalized variable associated with the p-th term, and C_{uv} represents the interaction or coupling between variables u and v, potentially incorporating the p-th penalty criterion to control regularization or complexity. This criterion is often used to balance model fitting with generalization, ensuring that overfitting is minimized by appropriately penalizing larger or more complex parameters.

- $a_u > 0$ for all u
- $b_v > 0$ for all v
- $C_{uv} \geq 0$ for all u, v
- $\sum_{u=1}^{m} a_u = \sum_{v=1}^{n} b_v$

For the balanced linear transportation problem to have a workable solution, the balanced condition is regarded as a necessary and sufficient condition. A typical transportation problem consists of exactly $m + n$ variables and $m + n$ constraints.

The transportation problem is solved like a typical linear programming problem because the LINDO software handles the problem in explicit equation form.

An Exponential Membership Function

According to traditional fuzzy set theory, a membership function gives each element in the discourse universe a value between 0 and 1, indicating how much the element belongs to a particular set. One minus the membership degree gives the degree of non-membership, which is simply the complement of the membership value [8]. In practice, however, when someone states how much an element belongs to a fuzzy set, they frequently fail to provide a comparable degree of non-membership to complement this value of 1.

The exponential membership function is defined as:

$$\mu_p^{EZ}(x) = \begin{cases} 1 & \text{if } Z_p \leq L_p \\ \frac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}} & \text{if } L_p \leq Z_p \leq U_p \\ 0 & \text{if } Z_p \geq U_p \end{cases} \quad (2.5)$$

Where:

- $Z_p(x)$ represents a function or transformation of the input variable x at the p-th instance.
- L_p and U_p are constants associated with the p-th instance, typically denoting lower and upper bounds respectively.
- The equation normalizes $Z_p(x)$ to a dimensionless form $\psi_p(x)$, scaling the values typically to a $[0,1]$ range, assuming $Z_p(x) \in [L_p, U_p]$.
- s is a non-zero parameter determined by the decision maker.

The parameter $p = 1,2,3,\dots,P$ indexes a set of such transformations, potentially across multiple dimensions, time steps, or data points.

Triangular Fuzzy Number (TFN)

A fuzzy number $A = (a, b, c)$ is said to be a Triangular Fuzzy Number (TFN) if its membership function is defined as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } x = b \\ \frac{c-x}{c-b} & \text{for } b \leq x \leq c \\ 0 & \text{otherwise} \end{cases} \quad (2.6)$$

Pentagonal Fuzzy Number (PFN)

A Pentagonal Fuzzy Number (PFN) is characterized by five key points: Left end a_1, left mode a_2, middle mode a_3, right mode a_4, and right end a_5.

Its membership function $\mu_A(x)$ is defined as:

$$\mu_A(x) = \begin{cases} 0, & \text{if } x \le a_1 \\ \frac{\alpha}{2} \cdot \frac{(x-a_1)}{(a_2-a_1)}, & \text{if } a_1 \le x \le a_2 \\ \frac{\alpha}{2} + \frac{\alpha}{2} \cdot \frac{(x-a_2)}{(a_3-a_2)}, & \text{if } a_2 \le x \le a_3 \\ \frac{\alpha}{2} + \frac{\alpha}{2} \cdot \frac{(x-a_4)}{(a_3-a_4)}, & \text{if } a_3 \le x \le a_4 \\ \frac{\alpha}{2} + \frac{\alpha}{2} \cdot \frac{(x-a_5)}{(a_4-a_5)}, & \text{if } a_4 \le x \le a_5 \\ 0, & \text{if } x \ge a_5 \end{cases} \quad (2.7)$$

This structure creates a pentagonal (or trapezoidal-like) shape in the graph, where the membership rises to 1, stays flat, and then decreases back to 0.

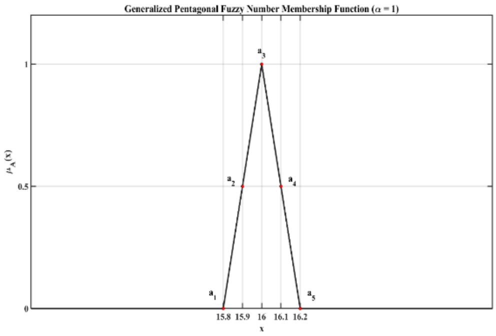

Figure 1: Pentagonal Fuzzy Number Membership Function

Let us assume that $R: F \to R$ Here, F represents the entire set of fuzzy numbers, and R is a linear-type function that converts each fuzzy number to its corresponding real number. Therefore, for any two parameters $\tilde{a}$ and $\tilde{b}$ we have:

$$\tilde{a} \ge \tilde{b} \Leftrightarrow \Re^*(\tilde{a}) \ge \Re^*(\tilde{b})$$

$$\tilde{a} > \tilde{b} \Leftrightarrow \Re^*(\tilde{a}) > \Re^*(\tilde{b})$$

$$\tilde{a} \equiv \tilde{b} \Leftrightarrow \Re^*(\tilde{a}) = \Re^*(\tilde{b})$$

(2.8)

Under this works focus is limited to clear ranking functions, defined as a ranking function $\Re^*$ as.

$$\Re^*\left(k\tilde{a} + \tilde{b}\right) = k\Re^*(\tilde{a}) + \Re^*\left(\tilde{b}\right) \quad (2.9)$$

for any $\tilde{a}, \tilde{b} \in F$ and any scalar $k \in \Re^*$.

2.2 A. Ranking function

When defuzzifying fuzzy logic systems, one technique used for ranking fuzzy numbers is Rouben's Ranking Function. Fuzzy controllers and other systems that must make decisions based on fuzzy logic frequently employ defuzzification, which is the act of turning a fuzzy set or fuzzy number into a crisp value. Rouben's approach uses a ranking index to help establish how "large" or "small" a fuzzy number is in relation to other fuzzy numbers, with the goal of ranking fuzzy numbers. Regarding fuzzy sets and their centroids, this function is helpful as it offers a quantitative method for defuzzification.

$$\Re^*(\tilde{a}) = \frac{1}{2}\int_0^1 \left(\inf\tilde{a}\alpha + \sup\tilde{a}\alpha\right)\, dx \quad (2.10)$$

We obtained the following results:

$$\Re^*(a) = \frac{1}{2}\left(a^L + a^U + \frac{1}{2}(\beta - \alpha)\right) \quad (2.11)$$

for trapezoidal fuzzy number $(a^L - \alpha, a^L, a^V, a^V + \beta)$.

For pentagonal fuzzy number (m, r_1, r_2, s_1, s_2), we have:

$$\Re^*(a) = \frac{S + 4m - R}{4} \quad (2.12)$$

where $S = s_1 + s_2$ and $R = r_1 + r_2$.

B. Solving Fuzzy Multi-Objective Transportation Problem (FMOTP)

An uncertain multi-objective transportation problem can be defined as follows. The preliminary fuzzy model for the problem (Equations 2.1–2.5) is:

Objective:
Find x_{uv}, where $u = 1,2,\dots,m$ and $v = 1,2,\dots,n$, such that:

$$Z_p \tilde{\le} L_p, \quad p = 1,2,3,\dots,P \quad (2.13)$$

Subject to:

$$\sum_{v=1}^{n} x_{uv} = a_u, \quad u \in \{1,2,\dots,m\} \quad (2.14)$$

$$\sum_{u=1}^{m} x_{uv} = b_v, \quad v \in \{1,2,\dots,n\} \quad (2.15)$$

$$x_{uv} \ge 0, \quad \text{for all } u, v \quad (2.16)$$

Here, the symbol $\tilde{\leq}$ denotes "fuzzified less than or equal to". The fuzzy cost coefficients $\tilde{a}_{uv}^{p}$ and $\tilde{c}_{uv}^{p}$ are given as pentagonal fuzzy numbers:

$$\tilde{a}_{uv}^{p} = (a_{uv}^{1}, a_{uv}^{2}, a_{uv}^{3}, a_{uv}^{4}, a_{uv}^{5})$$

$$\tilde{c}_{uv}^{p} = (c_{uv}^{1}, c_{uv}^{2}, c_{uv}^{3}, c_{uv}^{4}, c_{uv}^{5})$$

C. Definition: A solution $x \in X$ is considered feasible for the FMOTP (Equations 2.7–2.9) if it satisfies the constraints (2.8–2.9).

D. Definition: An optimal solution to the FMOTP for $x \in X$ is defined as follows:

$$\tilde{z}_u(x) \geq \tilde{z}_u(x^*) \quad \forall\, u = 1,2,\dots,q$$

That is, there exists no other $x \in X$ that satisfies this condition.

The FMOTP can be transformed into a classical multi-objective transportation problem (MOTP) using the ranking function $\mathfrak{R}^*$, as shown below:

$$\max R(\tilde{z}_p) = \sum_{v} R\left(\tilde{c}_{puv}\right) x_{uv}, \quad p = 1,2,\dots,q$$

subject to:

$$R(a_{uv}) x_{uv} \leq R(b_v), \quad u, v = 1,2,\dots,m$$

$$x_{uv} \geq 0, \quad \text{for all } u, v$$

$$\max z_p' = \sum_{v} c_{puv}\, x_{uv}$$

subject to:

$$\sum_{v} a_{uv}\, x_{uv} \leq b_v, \quad u, v = 1,2,\dots,m$$

$$x_{uv} \geq 0$$

Where $a_{uv}', b_u',$ and c_v' are real numbers corresponding to the fuzzy numbers $\tilde{a}_{uv}, \tilde{b}_u, \tilde{c}_v$ after applying the ranking function $\mathfrak{R}^*$.

3. FUZZY PROGRAMMING TECHNIQUE

Mathematically, MOTP can be solved as follows:

$$\min Z_p = \begin{cases} \sum_{u=1}^{m}\sum_{v=1}^{n} c_{uv}^{1}\, x_{uv} \\ \sum_{u=1}^{m}\sum_{v=1}^{n} c_{uv}^{2}\, x_{uv} \\ \\ \sum_{u=1}^{m}\sum_{v=1}^{n} c_{uv}^{p}\, x_{uv} \end{cases} \quad (3.1)$$

$$\sum_{v=1}^{n} x_{uv} = a_u, \quad u = 1,2,3,\dots,m. \quad (3.2)$$

Subject to:

$$\sum_{u=1}^{m} x_{uv} = b_v, \quad v = 1,2,3,\dots,n. \quad (3.3)$$

$$x_{uv} \geq 0 \quad \text{for all} \quad u, v.$$

We apply Zimmermann's fuzzy programming technique. The steps below provide a quick overview of the process.

Step 1. Employ a sequential optimization approach to address MOLPP. By isolating each objective function in turn and disregarding the others, solve the problem q times for q distinct objective functions. Denote the optimal solutions for these individual optimizations as $X_1, X_2, \dots, X_q$.

Step 2. Now that you have the q perfect solutions from Step 1, create a $q \times q$ payout matrix. Then, using the reward matrix, calculate the upper bound (U_p) and lower bound (L_p) for the objective function Z_p'. This yields inequality:

$$L_p \leq Z_p' \leq U_p \quad \text{for} \quad p = 1,2,\dots,q.$$

Step 3. The following is the formulation of an analogous crisp model for the fuzzy model if we use an exponential membership function as specified in (3.1). An exponential membership function is defined by:

$$\mu^E Z_p(x) = \begin{cases} 1 & \text{if} \quad Z_p \leq L_p, \\ \frac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}} & \text{if} \quad L_p \leq Z_p \leq U_p, \\ 0 & \text{if} \quad Z_p \geq U_p, \end{cases}$$

(3.4)

where

$$\psi_p(x) = \frac{Z_p(x) - L_p}{U_p - L_p}, \quad p = 1,2,\dots,P,$$

and s is a non-zero parameter prescribed by the decision-maker.

This is how an analogous crisp model for the fuzzy model is created if we apply the exponential membership function as specified in equation (3.4):

$$\text{Max}\lambda \quad (3.5)$$

subject to:

$$\lambda \leq \frac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}}, \quad p = 1,2,\dots,P. \quad (3.6)$$

$$\sum_{v=1}^{m} x_{uv} = b_v, \quad v = 1,2,\dots,n. \quad (3.7)$$

$$x_{uv} \geq 0 \quad \text{for all} \quad u, v. \quad (3.8)$$

The above problem (3.5–3.8) can be further simplified as:

$\max X_3$ (3.9)

subject to:

$s\{1-\psi_p(x)\} \geq X_3, \quad p = 1,2,\ldots,P.$

(3.10)

$\sum_{v=1}^{n} x_{uv} = a_u, \quad u = 1,2,\ldots,m.$ (3.11)

subject to:

$\sum_{u=1}^{m} x_{uv} = b_v, \quad v = 1,2,\ldots,n.$ (3.12)

$$x_{uv} \geq 0 \quad \forall u,v, \quad X_3 \geq 0,$$

where

$$X_3 = \log\{1+\lambda(e^s-1)\}.$$

Step 4. Find the best compromise solutions by solving the crisp model. Analyze the objective function values at the compromise decisions.

4. NUMERICAL EXAMPLE

According to the data from [3],

Objective Functions:

$$\min z_1(\mathbf{x}) = 16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33}$$

$$min\ z_1(\mathbf{x}) = 16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33}$$

subject to:

$$\sum_{v=1}^{3} x_{uv} = 14, \quad u = 1,2,3$$
$$\sum_{u=1}^{3} x_{uv} = 16, \quad v = 1,2,3$$
$$\sum_{u=1}^{3}\sum_{v=1}^{3} x_{uv} = 12$$
$$x_{uv} \geq 0, \quad u = 1,2,3, \quad v = 1,2,3.$$

Fuzzy Parameters:

$$\begin{aligned}
16 &= (15.8,15.9,16,16.1,16.2)\\
19 &= (18.8,18.9,19,19.1,19.2)\\
12 &= (11.8,11.9,12,12.1,12.2)\\
22 &= (21.8,21.9,22,22.1,22.2)\\
13 &= (12.8,12.9,13,13.1,13.2)\\
19 &= (18.8,18.9,19,19.1,19.2)\\
14 &= (13.8,13.9,14,14.1,14.2)\\
28 &= (27.8,27.9,28,28.1,28.2)\\
8 &= (7.8,7.9,8,8.1,8.2)\\
9 &= (8.8,8.9,9,9.1,9.2)\\
10 &= (9.8,9.9,10,10.1,10.2)\\
20 &= (19.8,19.9,20.1,20.2)\\
15 &= (14.8,14.9,15,15.1,15.2)\\
6 &= (5.8,5.9,6,6.1,6.2)\\
17 &= (16.8,16.9,17,17.1,17.2)\\
1 &= (0.8,0.9,1,1.1,1.2)
\end{aligned}$$

Fuzzy Objective Functions:

$$\begin{aligned}
\min \quad z_1(\mathbf{x}) = {}& \Re^*\left(\tilde{16}\right)x_{11} + \Re^*\left(\tilde{19}\right)x_{12} + \Re^*\left(\tilde{12}\right)x_{13}\\
&+\Re^*\left(\tilde{22}\right)x_{21} + \Re^*\left(\tilde{13}\right)x_{22} + \Re^*\left(\tilde{19}\right)x_{23}\\
&+\Re^*\left(\tilde{14}\right)x_{31} + \Re^*\left(\tilde{28}\right)x_{32} + \Re^*\left(\tilde{8}\right)x_{33}
\end{aligned} \tag{4.1}$$

$$\begin{aligned}
\min \quad z_2(\mathbf{x}) = {}& \Re^*\left(\tilde{9}\right)x_{11} + \Re^*\left(\tilde{14}\right)x_{12} + \Re^*\left(\tilde{12}\right)x_{13}\\
&+\Re^*\left(\tilde{16}\right)x_{21} + \Re^*\left(\tilde{10}\right)x_{22} + \Re^*\left(\tilde{14}\right)x_{23}\\
&+\Re^*\left(\tilde{8}\right)x_{31} + \Re^*\left(\tilde{20}\right)x_{32} + \Re^*\left(\tilde{6}\right)x_{33}
\end{aligned}$$

(4.2)

Fuzzy Constraints:

$$\begin{aligned}
&\Re^*\left(\tilde{1}\right)x_{11} + \Re^*\left(\tilde{1}\right)x_{12} + \Re^*\left(\tilde{1}\right)x_{13} = \Re^*\left(\tilde{14}\right)\\
&\Re^*\left(\tilde{1}\right)x_{21} + \Re^*\left(\tilde{1}\right)x_{22} + \Re^*\left(\tilde{1}\right)x_{23} = \Re^*\left(\tilde{16}\right)\\
&\Re^*\left(\tilde{1}\right)x_{31} + \Re^*\left(\tilde{1}\right)x_{32} + \Re^*\left(\tilde{1}\right)x_{33} = \Re^*\left(\tilde{12}\right)\\
&\Re^*\left(\tilde{1}\right)x_{11} + \Re^*\left(\tilde{1}\right)x_{21} + \Re^*\left(\tilde{1}\right)x_{31} = \Re^*\left(\tilde{10}\right)\\
&\Re^*\left(\tilde{1}\right)x_{12} + \Re^*\left(\tilde{1}\right)x_{22} + \Re^*\left(\tilde{1}\right)x_{32} = \Re^*\left(\tilde{15}\right)\\
&\Re^*\left(\tilde{1}\right)x_{13} + \Re^*\left(\tilde{1}\right)x_{23} + \Re^*\left(\tilde{1}\right)x_{33} = \Re^*\left(\tilde{17}\right)\\
&x_{uv} \geq 0, \quad u = 1,2,3, \quad v = 1,2,3.
\end{aligned} \tag{4.3}$$

Equivalent Crisp Model:

$$\begin{aligned}
\min \quad z_1(\mathbf{x}) = {}& 15.9x_{11} + 18.9x_{12} + 11.9x_{13}\\
&+21.9x_{21} + 12.9x_{22} + 18.9x_{23}\\
&+13.9x_{31} + 27.9x_{32} + 7.9x_{33}
\end{aligned}$$

(4.4)

$$\min \quad z_2(\mathbf{x}) = 8.9x_{11} + 13.9x_{12} + 11.9x_{13} + 15.9x_{21} + 9.9x_{22} + 13.9x_{23} + 7.9x_{31} + 19.9x_{32} + 5.9x_{33} \tag{4.5}$$

Subject to:

$$\begin{aligned} 1.01x_{11} + 1.02x_{12} + 1.01x_{13} &= 14.9 \\ 1.01x_{21} + 1.01x_{22} + 1.02x_{23} &= 15.9 \\ 1.02x_{31} + 1.01x_{32} + 1.01x_{33} &= 11.9 \\ 1.02x_{11} + 1.01x_{21} + 1.01x_{31} &= 9.9 \\ 1.01x_{12} + 1.02x_{22} + 1.01x_{32} &= 14.9 \\ 1.01x_{13} + 1.02x_{23} + 1.01x_{33} &= 16.9 \\ x_{ij} \geq 0, \quad i = 1,2,3, \quad j &= 1,2,3. \end{aligned} \tag{4.6}$$

Solution:

Using step 1 and step 2, minimize z_1 subject to constraints (4.6) as:

$$x_{13} = 14.752, \quad x_{22} = 15.742, \quad x_{33} = 11.782$$

with $Z_R(X_1) = 456, Z_R(X_2) = 471$.

Similarly, minimize z_2 subject to constraints (4.6) as:

$$x_{11} = 14.752, \quad x_{22} = 15.742, \quad x_{33} = 11.782$$

with $Z_C(X_1) = 320, Z_C(X_2) = 356$.

Using step-3 the exponential membership function and formulate the equivalent crisp model for the fuzzy problem. Apply the constraints using the bonds identified in step 2 and solve the resulting optimization problem.

$U_1 = 471, L_1 = 456, U_2 356, L_2 = 320$. Find x_{ij}, $i = 1,2,3; j = 1,2,3$. So as to satisfy:

$Z_R \leq 471, Z_C \leq 320$ constraints (4.6). An equation crisp model with the parameter $s = 1$ can be stated as:

$$\min X_3 \tag{4.7}$$

subject to:

$$s[z_1(x)] + X_4(U_1 - L_1) \geq s(U_1)$$

$$s[z_2(x)] + X_4(U_2 - L_2) \geq s(U_2) \tag{4.8}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2, \dots, m \tag{4.9}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2, \dots, n \tag{4.10}$$

$$x_{ij} \geq 0 \quad \forall i,j, \quad X_3 \geq 0$$

where $X_3 = \log\{1 + \lambda(e^s - 1)\}$.

$$\min \quad z_1(\mathbf{x}) = 15.9x_{11} + 18.9x_{12} + 11.9x_{13} + 21.9x_{21} + 12.9x_{22} + 18.9x_{23} + 13.9x_{31} + 27.9x_{32} + 7.9x_{33} + 15X_4 \geq 471 \tag{4.11}$$

$$\min \quad z_2(\mathbf{x}) = 8.9x_{11} + 13.9x_{12} + 11.9x_{13} + 15.9x_{21} + 9.9x_{22} + 13.9x_{23} + 7.9x_{31} + 19.9x_{32} + 5.9x_{33} + 35.887X_4 \geq 356 \tag{4.12}$$

subject to:

$$\begin{aligned} 1.01x_{11} + 1.02x_{12} + 1.01x_{13} &= 14.9 \\ 1.01x_{21} + 1.01x_{22} + 1.02x_{23} &= 15.9 \\ 1.02x_{31} + 1.01x_{32} + 1.01x_{33} &= 11.9 \\ 1.02x_{11} + 1.01x_{21} + 1.01x_{31} &= 9.9 \\ 1.01x_{12} + 1.02x_{22} + 1.01x_{32} &= 14.9 \\ 1.01x_{13} + 1.02x_{23} + 1.01x_{33} &= 16.9 \\ x_{uv} \geq 0 \quad \text{for all} \quad u, v = 1,2,3 \quad \text{and} \quad X_3 &\geq 0. \end{aligned} \tag{4.13}$$

The best possible solution to the given problem i.e.

$$\begin{cases} x_{11} = 14.752, \quad x_{22} = 15.742, \quad x_{33} = 11.782 \\ x_{31} = 9.801, \quad x_{13} = 14.752 \\ \text{Remaining } x_{uv}\text{'s are zero.} \end{cases}$$

with

$$Z_1 = 456.8768, \quad Z_C X_2 = 356.6634, \quad \lambda = 0.50.$$

LINGO program's Output

Global optimal solution found.

Objective value : 356.6634

Infeasibilities : 0.000000

Total solver iterations : 0

Elapsed runtime seconds: 0.07

Model Class: LP

Total variables : 11

Nonlinear variables : 0

Integer variables : 0

Total constraints : 5

Nonlinear constraints : 0

Total non zeros : 20

Nonlinear non zeros : 0

Variable	Value	Reduced Cost
X11	14.75248	0.000000
X12	0.000000	4.911881
X13	0.000000	3.000000
X21	0.000000	6.000000
X22	15.74257	0.000000
X23	0.000000	3.901980
X31	0.000000	1.532673
X32	0.000000	14.00000
X33	11.78218	0.000000
X4	0.000000	35.88700
MIN	356.0000	0.000000

Row	Slack or Surplus	Dual Price
1	356.6634	-1.000000
2	0.000000	0.000000
3	0.000000	-8.811881
4	0.000000	-9.801980
5	0.000000	-5.841584

Figure 2. Objective value Z1 obtained by LINGO Software

Global optimal solution found.

Objective value : 456.8768

Infeasibilities : 0.000000

Total solver iterations: 0

Elapsed runtime seconds: 0.04

Model Class: LP

Total variables : 11

Nonlinear variables: 0

Integer variables : 0

Total constraints : 5

Nonlinear constraints : 0

Total non zeros : 20

Nonlinear non zeros: 0

Variable	Value	Reduced Cost
X11	0.000000	1.862376
X12	0.000000	6.126471
X13	0.000000	4.000000
X21	0.000000	8.000000
X22	14.60784	0.000000
X23	0.000000	10.92178
X31	9.801980	0.000000
X32	0.000000	15.12647
X33	16.73267	0.000000
X4	0.000000	15.00000
MIN	471.0000	0.000000

Row	Slack or Surplus	Dual Price
1	456.8768	-1.000000
2	0.000000	0.000000
3	0.000000	-13.76238
4	0.000000	-12.64706
5	0.000000	-7.821782

Figure 3. Objective value ZcX2 obtained by LINGO Software

5. Conclusions

Finally, this work provides a simple method for applying the Zimmermann methodology with exponential membership functions to convert a FMOTP into a crisp one. The suggested approach makes it easier to solve multi-objective fuzzy transportation issues in an efficient manner, especially when working with pentagonal fuzzy numbers. The study illustrates how fuzzy numbers can accurately simulate real-world issues with imprecise parameters by contrasting the results produced from hyperbolic and pentagonal membership functions with those obtained using exponential membership functions. The suggested technique provides a workable substitute for solving FMOT issues by streamlining the application and lowering computing overhead.

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