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Effect Of Variable Suction On Oscillatory Flow

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Abstract: In the present study, we considered two dimensional flow of water an infinite vertical porous plate with certain assumptions and constant temperature. The expression for transient velocity and transient temperature have been shown. The variation of different parameters amplitude and phase of skin friction, amplitude of rate of heat transfer, phase of skin friction, transient velocity and transient temperature are shown by Graphs.

Keywords: Suction Parameter, Transient Velocity, Heat Transfer , Transient Temperature.

Introduction: An analysis of a two-dimensional flow of water past an infinite vertical porous plate is presented under the following conditions:

(1) The suction velocity oscillates in time about a constant non- zero mean.

(2) The free stream velocity oscillates in time about a constant mean.

(3) The plate temperature is constant.

(4) The difference between the temperature of the plate and the free stream is moderately large causing free convection currents.

Approximate solutions for the coupled non-linear equations are obtained for the transient velocity, the transient temperature, the amplitude and the phase of the skin-friction and the rate of heat transfer. During the course of discussion, the effects of +G (The Grashof number, G>0 cooling of the plate by the free convection current G < 0 heating of the plate by the free convection currents), A variable suction parameter and frequency have been discussed. The flow past an infinite vertical porous and isothermal plate with constant suction was studied by Soundalgekar {1973}.The effects of the free convection currents on the mean flow were discussed and those on the unsteady flow were discussed in {1973}. The plate was assumed to be stationary. For the plate moving in its own plane, the effects of free convection currents, on the flow, were studied by Soundalgekar and Gupta (1974) in case of constant suction. The effects of variable suction on the flow past a vertical, stationary, isothermal plate were studied by Soundalgekar {1977}. In the above mentioned papers, it was assumed that the flow is of air or water at normal temperature and atmospheric pressure. But the behavior of the water at 4°C in different from that at normal temperature and pressure. Under normal conditions, the difference between the density is a linear function of the difference between the temperature at two specific points which is defined as

$$\Delta\rho = -\rho B \,(\Delta T)$$

(1)

Where β is the coefficient of the thermal expansion. But this analysis is not applicable to the study of the flow of water at 4°C past a vertical plate. This is because at 4°C the density of water is a maximum at atmospheric pressure and the above relation (1) does not hold good. The modified form of (1) applicable to water at 4°C is given by

$$\Delta\rho = -\rho\gamma\left(\Delta T\right)_2$$

(2)

Where γ = 8 x 10^{16} $K^{-2.}$ Taking this fact into account the free convection effects on the oscillatory flow of water at 4°C past an infinite vertical porous plate with constant suction were presented by Soundalgekar {1973}. It is now proposed to study the effects of variable suction on the oscillatory flow of water at 4°C past an infinite vertical porous plate. Yukio Sudo et al {1987} , Rudraiah et al. {1980}, Fetecau (2003), Chen and Tsuromo {1980} studied the combined free and forced convections heat transfer along plates.

Mathematical Analysis: We assume the unsteady flow of water at 4°C past an infinite, porous, vertical plate in the upward direction. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. The fluid properties are assumed constant. Then under usual Boussinesq's approximation and using equation (2), by Soundalgekar (1973) that the problem is now governed by the following equations.

$$\frac{\partial V'}{\partial Y'} = 0 \tag{3}$$

$$\rho' = \left(\frac{\partial u'}{\partial t'} + \nu \frac{\partial u'}{\partial y'}\right) = \rho^{\frac{\partial U'}{\partial t'}} + E_x \rho' \nu \left(T - \tau'_\infty\right)^2 + \mu \frac{\partial^2 u'}{\partial y'^2} \tag{4}$$

$$\rho' c_\rho = = \left(\frac{\partial T'}{\partial t'} + \nu \frac{\partial T'}{\partial y'}\right) = k' \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'}\right)^2 \tag{5}$$

With the help of following boundary condition

$$u' = 0, \quad T' = T'_w \quad \text{at} \quad y' = 0$$

$$u' = U'(t), \; T' = T'_w \quad \text{at} \quad y' \to \infty \tag{6}$$

For variable suction, integrating equation (5) and using boundary condition (6), we have

$$V' = -V_0\left(1 + \varepsilon A e^{i\omega' t'}\right) \tag{7}$$

Where $\varepsilon << 1$ and ε A < 1, and the negative sign in equation (5) indicates that the suction is directed towards the plane. Defining the non-dimensional quantities as notation, equations (3) to (5) and using boundary condition 6, we get the expression for velocity in view if equation (7) reduces to following non-dimensional form.

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial U}{\partial y} + G\theta^2 + \frac{\partial^2 u}{\partial y^2} \tag{8}$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - P\left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

And the boundary conditions are

$$u = 0, \quad \theta = 1 \quad \text{at} \quad y = 0 \tag{10}$$

$$u = U(t), \; \theta = 1 \quad \text{at} \quad y \to \infty$$

Equations (8) and (9) are the coupled non-linear equations and cannot be solved in exact from. So we now seek to find approximate solutions. Assuming the amplitude of the free stream oscillations to be small less than1, following by Light hill {1954}, we now assume for the velocity and temperature field in the neighborhood of this plate.

$$u(y,t) = u_\theta(y) + \varepsilon e^{i\omega t} u_1(y)$$ (11

$$\theta(y,t) = \theta_\theta(y) + \varepsilon e^{i\omega t} \theta_1(y)$$ (12)

And for free stream

$$U(t) = 1 + \varepsilon e^{i\omega t},$$ where $\varepsilon <<< 1$

Now substituting equations (11) and (12) in equations (8), (9) and using boundary condition (10) and equating the harmonic and non-harmonic terms, we get the following set of coupled non-linear equations.

$$u_0^\pi + u_0^\tau = G\theta_0^2$$ (13)

$$u_0^\pi + u_0^\tau - \frac{i\omega}{4} u_1 = Au_0' - \frac{i\omega}{4} - 2G\theta_0\theta_1$$ (14)

$$\theta_0^\pi + P\theta_0' = PEu_0'^2$$ (15)

$$\theta_0' P \theta_1' - \frac{i\omega}{4}\theta_1 = -PAu_0'u_1'$$ (16)

Here the prime denotes the differentiations with respect to y. The corresponding boundary conditions are

$$u_0 = 0,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 0$$ at $y = 0$

(17)

$$u_0 = 0,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 0$$ at $y \to \infty$

To solve these coupled non-linear equations, we now assume that the heat due to viscous dissipation is superimposed on the motion. Mathematically, this can be achieved by expanding the velocity and temperature in powers of E and the Eckert number, in case of incompressible fluids, E is always very small (<< 1). Hence we now assume.

$$u_0(y) = u_{01}(y) + Eu_{02}(y) + O\left(E^2\right)$$

$$\theta_0(y) = \theta_{01}(y) + \theta u_{02}(y) + O\left(E^2\right)$$ (18)

$$u_1(y) = u_{11}(y) + Eu_{12}(y) + O\left(E^2\right)$$

$$\theta_1(y) = \theta_{11}(y) + \theta u_{12}(y) + O\left(E^2\right)$$

Substituting equation (18) in equations (13) - (16) and using boundary condition (17) ,equating to zero the coefficients of different powers of E and neglecting the terms of 0(E^2) ,we obtained the following set of equations.

$$u_{01}^n + u_{01}' = -G\theta_{02}^2$$ (19)

$$u_{02}^n + u_{02}' = 2G\theta_{02}$$ (20)

$$u_{11}^\pi + u_{11}^\tau - \frac{i\omega}{4} u_{11} = Au_{01}' - \frac{i\omega}{4} - 2G\theta_{01}\theta_{11}$$ (21)

$$u_{12}^{\pi} + u_{12}^{\tau} - \frac{i\omega}{4} u_{12} = Au_{02}' - 2G(\theta_{01}\theta_{12} + \theta_{11}\theta_{02})$$
(22)

$$\theta_1' + P\theta_{01}' = 0$$
(23)

$$\theta_{02}'' + P\theta_{02}' = -Pu_{02}'^2$$
(24)

$$\theta_{11}'' + P\theta_{11}' - \frac{i\omega P}{4}\theta_{11} = -PA\theta_{01}'$$
(25)

$$\theta_{12}'' + P\theta_{12}' - \frac{i\omega P}{4}\theta_{12} = -PA\theta_{02}' - 2Pu_{01}'u_{02}'$$
(26)

The corresponding boundary conditions are

$$u_1^0(0) = 0,\ u_2^0(0) = 0, u_{11}(0) = 0,\ u_{12}(0) = 0$$

$$\theta_1^0(0) = 0,\ \theta_2^0(0) = 0, \theta_{11}(0) = 0,\ \theta_{12}(0) = 0$$

$$u_1^0(\infty) = 0,\ u_2^0(\infty) = 0, u_{11}(\infty) = 0,\ u_{12}(\infty) = 0$$
(27)

$$\theta_1^0(\infty) = 0,\ \theta_2^0(\infty) = 0, \theta_{11}(\infty) = 0,\ \theta_{12}(\infty) = 0$$

Equations (19) - (24) are now coupled linear equations. They are solved as follows. First equation (23) is solved for at under its boundary conditions and its value is substituted in equation (19) and is solved next. This is repeated and we obtained the solutions for equations (19) - (26). The method being straight forward, the solutions are not mentioned here to save space.

We can now express the expressions for the velocity and the temperature field given in equations (11) and (12) respectively, in terms of the fluctuating parts as

$$u = u_0 + \varepsilon(M_1 \cos\omega t - M_1 \sin\omega t)$$
(28)

$$\theta = \theta_0 + \varepsilon(T_1 \cos\omega t - T_1 \sin\omega t)$$
(29)

Where Hence, we can now derive expressions for the transient velocity and the transient temperature from equations (28) and (29) by putting $\omega t = \frac{\pi}{2}$.

$$u = u_0 - \varepsilon M_1'$$

$$\theta = \theta_0 - \varepsilon T_1'$$
(30)

The numerical calculations for the transient velocity u and the transient temperature θ have been carried out for different values of G, E and A. They are shown on Figure 1 & Fig 3. We observe that from this Figure 1 that an increase in the suction parameter A or an increase in the Grashof number G leads to an increase in the transient velocity or the transient temperature. But an increase in A leads to an increase in the transient velocity and a decrease in the value of the transient temperature.

$$\tau = \frac{\tau'}{\rho U_0 V_0} = \left(\frac{du}{dx}\right)_{y=0}$$

$$\tau = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon e^{i\omega t}\left(\frac{du_1}{dy}\right)_{y=0}$$
(31)

The first term $\left(\frac{du_0}{dy}\right)_{y=0}$ is the mean skin-friction and has been discussed by Soundalgekar {1977}. As we are interested in knowing the Effects of A, G and ω on the amplitude and the phase of the skin- friction, we express equation (32) in terms of the amplitude and the phase as.

$$\tau = \left(\frac{du_0}{dy}\right)_{y=0} + E|B|\,Cos(\omega t + \alpha)$$

(32)

Where $B = B_\tau + iB_i = \left(\frac{du}{dy}\right)_{y=0}$

, and $\tan\alpha = \frac{B_i}{B_\tau}$ (33)

We now study the rate of heat transfer and it is defined as

$$q' = -k\left(\frac{dT'}{dY'}\right)_{y'=0}$$

(34)

Which reduces to following non-dimensional form?

$$q = \frac{q'V}{V_0 k\left(T' - T'_\omega\right)} = \left(\frac{d\theta}{dy}\right)_{y=0}$$

$$q = \left(\frac{d\theta}{dy}\right)_{y=0} + \varepsilon e^{i\omega t}\left(\frac{d\theta_1}{dy}\right)_{y=0}$$

(35)

This can be expressed in terms of the amplitude and the phase as

$$q = \left(\frac{d\theta}{dy}\right)_{y=0} + \varepsilon|Q|\,Cos(\omega t + \beta)$$

$$Q = Q_\tau + iQ_i = \left(\frac{d\theta_1}{dy}\right)_{y=0},$$ where $\tan\beta = \frac{Q_i}{Q_1}$

(36)

Results and Discussions: The numerical values of $|B|$ and $\tan\alpha$ have been calculated and they are shown in figure 5. It was observed by Soundalgekar {1977} that the amplitude B of the skin-friction increases with increasing ω, when the suction is constant. But in the presence of variable suction, for water at 4°C, the amplitude of the skin-friction has been observed to decrease with increasing ω. Also an increase in A, G or E leads to an increase in the value of $|B|$. $|Q|$ is shown in Figure 2. It has been observed by Soundalgekar (1977), that in the presence of the constant suction, for water at 4°C, the amplitude $|Q|$ of the rate of heat transfer increases with increasing G but in the presence of variable suction, $|Q|$ decreases with increasing G or E. However, $|Q|$ increases with increasing A. An increase ω in always leads to a decrease in the value of $|Q|$.

In Figure 5 & 6, Soundalgekar {1977} was observed that for constant suction, there is always a phase-lag. But in the present case for both the skin-friction and the rate of heat transfer, we observe that there is always a phase-lag.

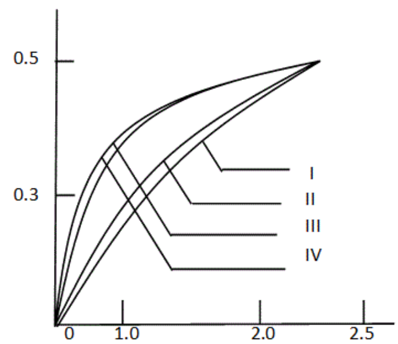

Fig 1: Transient Velocity Profile

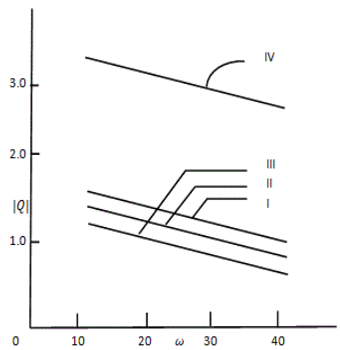

Fig 2: Amplitude of Rate of Heat Transfer

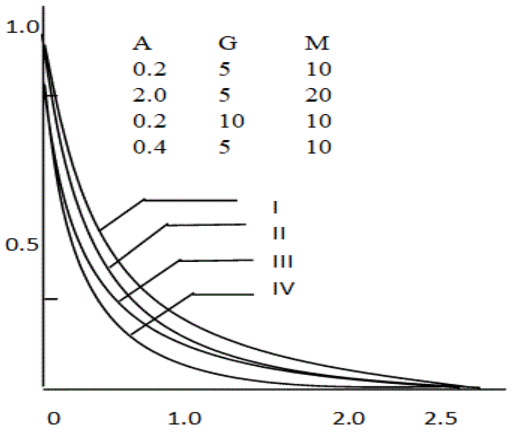

Fig 3: Transient Temperature profile

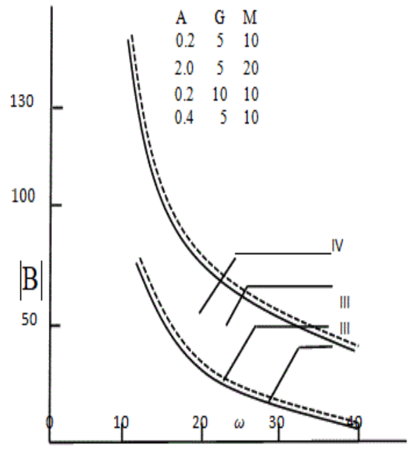

Fig 4: Amplitude of Skin Friction

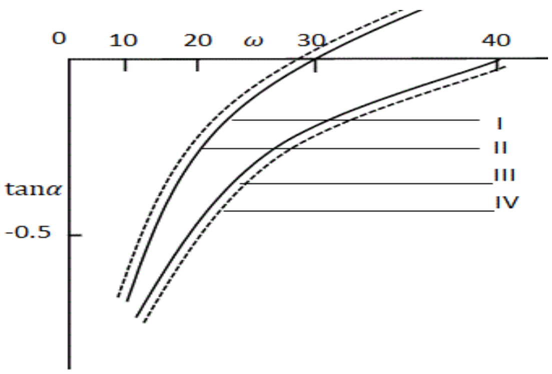

Fig 5: Phase of Skin Friction

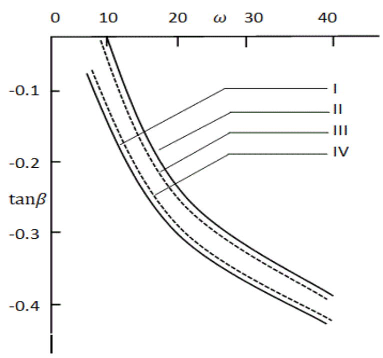

Fig 6: Phase of rate of Heat Transfer

Conclusion:

1. The transient velocity or the transient temperature increases with increasing A or G.

2. Due to increasing A , the transient velocity increases.

3. There is a fall in the transient temperature owing to an increase in A.

4. In the presence of a variable suction, $|B|$ decreases with increasing ω. But $|B|$ increases with increasing A, G, or E.

5. In the presence of a variable suction, $|Q|$ decreases with increasing G or E and increasing with increasing A. But with increasing E, $|Q|$ always decreases.

6. In the present of a variable suction, there is always a phase lag for both the skin-friction and the rate of heat transfer.

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