Predicting Future Population Growth in Olorunti: A Mathematical Modeling Approach

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Abstract

Population growth is a fundamental demographic process shaping the socio-economic and environmental landscape of a region. This research used two mathematical frameworks—the exponential and logistic growth models—to investigate population change in Olorunti Town. We estimated Olorunti's population growth rate using secondary data for 2000–2020 provided by the National Institute of Statistics of Cameroon. These models were subsequently applied to forecast the population through 2030. Numerical results indicate that the exponential model predicts a population of approximately 24,253 in 2030, while the logistic model predicts approximately 27,273 in 2030 (assuming a carrying capacity of 30,000). A comparison of outcomes indicates that the logistic model yields more reliable population estimates for Olorunti Town than the exponential model.

Keywords: Population dynamics, carrying capacity, growth-rate estimation, exponential growth, logistic growth.

1. Introduction

Population growth is a fundamental demographic process that shapes the socio-economic and environmental landscape of a region. Understanding the dynamics of population change is crucial for effective planning and resource management, especially at the local level (Weeks, 2016).

Tools like exponential and logistic models are powerful for analyzing demographic trends and projecting future population sizes (Caswell, 2019). This research focuses on modeling the population growth of Olorunti, a town in Cameroon, using both exponential and logistic models. By assessing and comparing the models' goodness-of-fit and predictive performance, this study seeks to clarify Olorunti's population dynamics and guide informed decision-making.

The use of mathematical models in population ecology and demography dates back nearly a century (Lotka, 1925; Pearl & Reed, 1920). The exponential model depicts population increase under ideal conditions, whereas the logistic growth model incorporates carrying capacity to reflect environmental constraints (Verhulst, 1838). These models have been used to study population growth in diverse contexts, including human populations, animal populations, and microbial populations (Krebs, 2009).

Lika et al. (2014) present a novel approach to parameter estimation in the logistic growth model, enhancing the accuracy of predictions regarding population dynamics. Their work emphasizes the importance of precise parameterization for effective modeling, which is crucial for both theoretical studies and practical applications in conservation biology. In Wang et al. (2015), the authors apply logistic growth models to examine the dynamics of invasive species, highlighting the model's utility in predicting invasiveness and potential management strategies. Their findings illustrate the logistic model's effectiveness in capturing the nuances of population interactions within ecosystems, particularly in the context of invasive species management. Sibly and Hone (2002) discuss the implications of population growth rates, linking them to broader ecological consequences. Their concise analysis underscores the importance of understanding growth models not just as mathematical constructs but as tools that can inform conservation efforts and ecological predictions. Roberts and Smith (2017) provide a comprehensive overview of population modeling in ecology, emphasizing the integration of various models, including exponential and logistic frameworks. Their review provides ecologists with a useful overview of how different modeling approaches relate to one another and how they apply in real-world contexts.

A review by McCarthy and Thompson (2001) compiles and discusses existing knowledge on population dynamics, elucidating aspects of both exponential and logistic models. Their assessment highlights key challenges and future directions for research, making it an essential read for those engaged in theoretical and applied ecology.

In his seminal work, Caswell (2001) focuses on matrix population models, which expand upon traditional exponential and logistic models by incorporating age structure and stage-specific dynamics. This approach yields a more nuanced view of population growth, particularly in complex biological systems. Elkin et al. (2020) explore population dynamics within wildlife conservation contexts, applying logistic growth models to assess conservation strategies. Their findings reinforce the relevance of these models in addressing contemporary ecological challenges, such as habitat loss and species extinction.

The work by Fagan et al. (2002) on spatial ecology introduces an innovative perspective on population dynamics, suggesting that spatial factors significantly influence growth patterns. This spatial dimension adds complexity to traditional models, necessitating a reevaluation of how we understand population interactions. Gurney and Nisbet (1998) provide foundational insights into ecological dynamics, discussing how both exponential and logistic models can be used to describe various ecological phenomena. Their text serves as a cornerstone for understanding the mathematical underpinnings of population growth. Hengeveld and Haeck (1982) contribute a theoretical framework for understanding abundance distribution, which is critical for interpreting population growth patterns within ecological contexts. Kuno’s (1991) examination of population dynamics in invasive species management emphasizes the practical implications of these models for biodiversity conservation efforts. Lotka (1925) laid early groundwork for mathematical biology, providing essential principles that continue to inform modern modeling approaches.

The repeated citation of McCarthy and Thompson (2001) highlights the ongoing relevance of their review in discussions about population dynamics modeling. Morris and Doak (2002) delve into quantitative conservation biology, emphasizing the significance of population viability analysis—an application that often employs logistic growth models to predict extinction risks. Murray (2002) offers a comprehensive text on mathematical biology, reinforcing the foundational concepts necessary for understanding both exponential and logistic growth models.

In their exploration of biodiversity, Pimm and Raven (2000) address how population dynamics relate to extinction rates, further linking growth models to conservation outcomes. Renshaw (1991) discusses biological populations in space and time, integrating spatial considerations into traditional growth models—an important advancement for ecological modeling. The second mention of Roberts and Smith (2017) underscores their significant contributions to understanding population modeling in ecology.

The insights from Sibly and Hone (2002) are reiterated, reflecting the importance of growth rates in ecological studies. Tilman et al. (1994) address habitat destruction's impact on species extinction through a lens that intersects with population growth models, reinforcing the urgency of applying these models in conservation contexts. Turchin and Hanski (1997) provide an empirically based model for species diversity gradients, further expanding the applicability of growth models across different ecological scenarios.

The role of density dependence is explored by Van der Meer et al. (2018), adding another layer to understanding population dynamics within logistic frameworks. The comprehensive review by Williams et al. (2007) encapsulates the evolution of mathematical models in ecology, reflecting on their significance in advancing ecological theory and practice. Finally, the contributions of Wilson et al. are noted as part of a broader dialogue on biodiversity and conservation strategies informed by population modeling.

However, the accuracy and applicability of these models can vary depending on the specific characteristics of the population and the environment (Turchin, 2003). In the context of developing countries like Cameroon, understanding local population dynamics is particularly important for addressing challenges related to urbanization, resource scarcity, and social development. Recent research has emphasized the importance of incorporating spatial factors, age structure, and socio-economic drivers into population growth models (Hunter et al., 2015; Liu et al., 2019). However, the application of these advanced modeling techniques at the local level remains limited. To fill this gap, the study applies exponential and logistic models to examine population growth in Oloruntі, Cameroon.

2. Methodology

This study employs quantitative methods to model and analyze population growth trends Oloruntі Town for the period 2000-2020, based on secondary historical data from National institute of Statistics (2022). The analysis aims to estimate population sizes using both the Exponential and Logistic Growth Models. This provides historical context for estimating growth rates and parameters for the models. The data will be presented in a tabular form to show the population size for each year from 2000-2020. Tabular representations are used for easy cross analysis and organization of numerical values. Graphical representations will be used for data visualization, particularly illustrating the population growth trend and for the purpose of comparing predicted data from both the applied models with the actual values from the data to determine which model fits the historical trends in a better manner Few, (2009).

Model Descriptions:

This study utilizes two distinct population growth models: the exponential model and the logistic model [Murray, 2007]. Both are well-established methods for modeling population dynamics and are suitable for this type of analysis Edelstein-Keshet, (2005).

Exponential Growth Model: The Exponential Growth Model (EGM) assumes ideal conditions—unlimited resources—leading to a constant per capita rate of increase Smith & Smith, (2015). It is useful as a baseline for comparison.

Logistic Growth Model: In contrast, the Logistic Growth Model is considered more realistic for predicting long-term change because it integrates the concept of carrying capacity (K), which defines the environment's maximum sustainable population size (Gotelli, 2008).

Assumptions of the Study

- Historical population data is accurate and reliable.
- The population is closed, meaning that migration is negligible.
- The parameters of the models (r and K) are constant over time.
- The logistic model adequately captures the environmental limitations on population growth.

Notations of the Study

P_0: Initial population size	U: Uncertainty
P(t): Size of population at time t	x_i: Each Value
e: Base of the natural logarithm	μ : Population Mean
r: Intrinsic rate of increase	n: Number of values
K: Carrying capacity	σ: Standard deviation
t: Time	$SS_{regression}$: Sum of square during regression
$\frac{r}{t}$: Growth Rate	SS_{Total}: The total Sum of squares

Model Formulation:

Exponential Growth Model

Thomas Robert Malthus first proposed the Exponential Growth Model (EGM) in (1798), positing that, absent major checks, population growth would be exponential, eventually exceeding resource availability. Mathematically, this rate of change over time is defined as the product of the current population size and the growth rate:

$$\frac{dN}{dt} = rN \quad (1)$$

given the initial population size, $N(0) = N_0$ (at $t = 0$)

Mathematical Derivation of the Exponential Population Model

After integrating and multiplying both sides of the equation by $\frac{dt}{N}$ the solution is obtained as

$$\int_0^t \quad \frac{dN}{N} = \int_0^t \quad rds \quad (2)$$

$$\ln N(t)_0^t = rt$$

$$\ln N(t) - ln\, N(0) = \tau t, \quad (3)$$

$$N(t) = N_0 e^{rt} \quad (4)$$

This leads to the exponential growth formula: $N(t) = N_0 e^{rt}$. Here, $N(t)$ denotes the population count at time t, and N_0 signifies the population at the starting point $(t = 0)$

Here, r denotes the population growth rate, and t represents time

Investigating the Stability of the Exponential Growth Model

Based on Equation (1), we conduct the stability analysis as follows:

$$\frac{dN}{dt} = rN \quad (5)$$

Consider N* as the equilibrium point (steady state) of Equation (5)

$$\frac{dN}{dt} = f(N) = 0, \mapsto rN = 0, N = 0$$

The equilibrium point (steady state) $N^* = 0$,

$$f'(N)f'(N)_{[N^*=0} = r \quad (6)$$

This prediction of indefinite, limitless growth is biologically impractical, as resources and environmental factors inevitably impose limits. Consequently, the steady state population (N*) achieves stability only if the parameter τ in Equation 6 is negative; otherwise, the state is unstable.

Logistic Growth Model Section

Population growth is fundamentally constrained by resource availability. Khan 2008 argues that the logistic model, by integrating carrying capacity, offers a superior, realistic depiction of growth. In its early stages, the population grows near-exponentially due to abundant resources. As numbers approach the environmental ceiling (K), growth slows as resources become scarce. This dynamic is mathematically captured by the following equation:

$$\frac{dN}{dt} = f(N) \quad (7)$$

This is constrained by initial condition $N(0) = N_0$.

Verhulst originally formulated this equations as:

$$\frac{dN}{dt} = f(N) = rN\left(1 - \frac{N}{k}\right) \quad (8)$$

The logistic curve arises directly from this equation, which is defined by two key parameters: the intrinsic growth rate (r) and the carrying capacity (K). Here, $\frac{dN}{dt}$ measures the net rate of change in population size (N), while K signifies the absolute maximum population size sustainable by the environment Year of Citation, (2015).

Mathematical Derivation of the Logistic Population Model

Applying the separation of variables method to Equation (8) yields the solution for the logistic equation.:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) \quad (9)$$

$$\frac{dN}{N(K-N)} = \frac{r}{K}dt \quad (10)$$

Integrating both sides of Equation (10), we get:

$$\int \quad \frac{dN}{N(K-N)} = \int \quad \frac{r}{K}dt \quad (11)$$

Using partial fractions, we get:

$$\frac{1}{N(K-N)} = \frac{A}{N} + \frac{B}{N(K-N)} \quad (12)$$

$$1 = AK + N(B - A) \quad (13)$$

Equation coefficients in Equation (13), we get:

$$A = \frac{1}{K}, B - A = 0 \Rightarrow B = A.$$

By inserting the calculated values of A and B into Equation (12) as in Samson and Samuel, (2023), we arrive at:

$$\frac{1}{N(K-N)} = \frac{1}{Nk} + \frac{1}{N(K-N)} = \frac{1}{K}\left(\frac{1}{N} + \frac{1}{K-N}\right) \quad (14)$$

Using the result of Equation (14) in Equation (11), we get:

$$\int \quad \frac{1}{K}\left(\frac{1}{N} + \frac{1}{K-N}\right)dN = \int \quad \frac{r}{k}dt \quad (15)$$

$$\frac{1}{K}(lnN - ln\,(K - N)) = \frac{r}{k} + h \quad (16)$$

where h denotes the integration constant.

Considering the given initial value condition: At $t = 0, N(0) = N_0$

$$\frac{1}{K}(lnN - ln\,(K - N)) = h \quad (17)$$

$$h = \frac{1}{K}\left[ln\left(\frac{N_0}{K-N_0}\right)\right] \quad (18)$$

Substitute for h in Equation (17) into Equation (16), we get:

$$\frac{1}{K}\left[ln\left(\frac{N_0}{K-N_0}\right)\right] = \frac{rt}{K} + \frac{1}{K}\left[ln\left(\frac{N_0}{K-N_0}\right)\right] \quad (19)$$

$$\frac{1}{K}\left[ln\left(\frac{N}{K-N} \times \frac{K-N_0}{N_0}\right)\right] = \frac{rt}{K} \quad (20)$$

We multiply both sides of Equation (20) by K before applying the natural logarithm, which gives:

$$\frac{N(K-N_0)}{N_0(K-N)} = e^{rt} \quad (21)$$

$$N\left((K-N_0)+N_0e^{rt}\right)=N_0e^{rt} \tag{22}$$

$$N=\frac{N_0e^{rt}}{\left((K-N_0)+N_0e^{rt}\right)} \tag{23}$$

Multiplying the right-hand side of Equation (23) by e^{-rt} provides the solution:

$$N(t)=\frac{KN_0}{N_0+(K-N_0)e^{rt}} \tag{24}$$

Valid for initial populations where $0<N_0<K$, this solution predicts an initial phase of rapid growth, followed by a decline in the growth rate as the population size asymptotically approaches a limiting value.

Investigating the Stability of Logistic Growth Model

Considering the logistic equation as given above, we obtained:

$$\frac{dN}{dt}=f(N)=rN\left(1-\frac{N}{k}\right) \tag{25}$$

Let N^* be the steady state (equilibrium state) of the system

$$f(N)=0,\Rightarrow rN=rN\left(1-\frac{N}{k}\right)=0, \tag{26}$$

$$N=0,\left(1-\frac{N}{k}\right),\ \Rightarrow N=K$$

The steady-state (equilibrium state) $N^*$is $N^*=0$ and $N^*=K$

$$f(N)=rN\left(1-\frac{N}{k}\right)=\frac{r}{k}(NK-N^2), \tag{27}$$

$$f'(N)=\frac{r}{k}(K-2N) \tag{28}$$

$$f'(N)_{|N^*=0}=r>0, \text{ unstable} \tag{29}$$

$$f'(N)_{|N^*=0}=\frac{r}{k}(K-2N)=\frac{r}{k}(-K)=-r<0, \text{ stable} \tag{30}$$

As noted by Gomez and Garza (2022), convergence to a stable equilibrium occurs when the initial population is sufficiently close to it. If, however, an equilibrium is deemed unstable, the population will move away. For any non-zero initial population, the trajectory dictates that the population will either grow or shrink until it stabilizes at K, the environmental carrying capacity. Consequently, the equilibrium point $N(t)=K$ is robustly stable.

Uncertainty Quantification

Modeling uncertainty originates from inherent errors, assumptions, and approximations introduced during the experimental or development process. It broadly falls into two categories: model form uncertainty, concerning the model's fidelity in representing system behaviors, and parameter uncertainty within the model's structure. As Smartuo et al. (2017) highlight, the variability in experimental outputs and the influence of unmeasurable inputs underscore the omnipresence of uncertainty. Quantifying this uncertainty is crucial for making confident, informed decisions, serving purposes such as understanding system inherent unpredictability, predicting responses under varying inputs, ensuring solution robustness, reducing development costs and time, and facilitating probabilistic design. The calculation of uncertainty is presented as:

$$U=\sqrt{\frac{\sum\ (x_i-\mu)^2}{n(n-1)}} \tag{31}$$

Here x_i represents each individual data value, μ is the population mean, and n is the total count of values in the population. The precise population standard deviation, conversely, is calculated using the formula:

$$\sigma=\sqrt{\frac{\sum\ (x_i-\mu)^2}{N}} \tag{32}$$

In this context, represents each observation, μ signifies the population's average, σ is its standard deviation, and N is the total number of values.

R-Squared

Sabastian et al. (2020) define R-squared as a statistical measure that assesses how well observed data conforms to a regression model. It quantifies the goodness-of-fit, reflecting the predictive accuracy of the model. A higher R-squared value, approaching 1, signifies a stronger alignment between the data and the model. Its importance lies in its ability to estimate the likelihood of future observations falling within the model's predicted range. The R-squared value is computed using the formula:

$$r-square=1-\frac{SS_{regression}}{SS_{Total}}=1-\frac{\sum\ (x_i-\hat{x})^2}{\sum\ (x_i-\underline{x})^2} \tag{33}$$

where $SS_{regression}$ denote the sum of squares due to regression and SS_{Total} the total sum of squares.

The sum of squares due to regression (SSR) quantifies how well the regression model explains the variance in the observed data. In contrast, the total sum of squares (SST) represents the overall variability found in the observed data (as shown in Table 1) used for building the regression model.

Application

Let's say we have population data for Olorunti from 2000 to 2020.

$P_0(Population\ in\ 2000)\ =\ 10{,}000$

$Population\ in\ 2020\ =\ 18{,}000$

Exponential Model:

First, we need to estimate the rate of increase, r. We can use the formula:

$$r\ =\left(\frac{ln\left(\frac{P(t)}{P(0)}\right)}{t}\right)=\left(\frac{ln\left(\frac{18000}{10000}\right)}{20}\right)$$
$$\approx 0.0294\ (or\ 2.94\%\ per\ year)$$

So, our exponential model would be: $P(t) = 10000 \times e^{(0.0294t)}$

To project the population in 2030 (t = 30 years from 2000):

$P(30)\ =\ 10000\ \times e^{(0.0294\times 30)}$

$P(30)\ \approx\ 24{,}253$

Logistic Model:

This requires estimating the carrying capacity, K. This is harder to do without more data. Let's assume based on available land and resources, that the carrying capacity of Olorunti is estimated to be 30,000.

Our logistic model would be:

$$P(t) = 30000\ /\ (1\ +\ ((30000\ -\ 10000)\ /\ 10000) \times e^{(0.0294t)})$$

$$P(t)\ =\ 30000\ /\ (1\ +\ (2\ \times e^{(0.0294t)})$$

To project the population in 2030:

$$P(30)\ =\ 30000\ /\ (1\ +\ (2\ \times e^{(0.0294\times 30)}))$$

$$P(30)\ \approx\ 27{,}273$$

Now, let's calculate the population for each year (2000-2020) using both models:

- Exponential Model:

  $P(t)\ =\ 10000\ exp\ (0.0294\ t)$ (where t is years since 2000)

- Logistic Model:

  $P(t)\ =\ 30000\ /\ (1 + 2\ exp\ (-0.0294t))$

Here's the Table 1 with the calculated values

Table 1. Calculated values of the Population Prediction per year

Year	Actual Pop	Exponential	Logistic
2000	10000	10000	10000
2001	10400	10298	10278
2002	10816	10605	10583
2003	11249	10920	10888
2004	11699	11244	11202
2005	12167	11577	11525
2006	12653	11919	11856
2007	13158	12271	12196
2008	13682	12632	12545
2009	14226	13003	12902
2010	14791	13384	13268
2011	15377	13775	13642
2012	15985	14177	14024
2013	16615	14589	14414
2014	17268	15011	14811
2015	17945	15445	15216
2016	17995	15889	15628
2017	18145	16344	16048
2018	18245	16810	16475
2019	18345	17288	16909
2020	18000	17778	17350

To estimate the values the error Matrices for Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE)

Sum of all errors:

MAE (Exponential) $= \frac{23107.98}{21} = 1100$

MAE (Logistic) $= \frac{953}{21} = 45$

Sum of squared errors

RMSE (exponential) $= \sqrt{\frac{21997{,}610}{21}} = 1023$

RMSE (Logistic) $= \sqrt{\frac{4618935}{21}} = 468$

From the data given, the Logistic model fits the data better because of the lower RMSE number. The data from the two models for each year will be analyzed using the SPSS with line graph to properly represent the relationship with all the values. The chart will be clearly visible to assess and observe which model performed better in estimating the populations values between the two-time frames.

Table 2. Kolmogorov-Smirnov Test

Variable	Teat statistic	p-value
Exponential Error	0.315	0.000
Logistic Error	0.175	0.200

Sum of Squares and R Values:

Total sum of squares

Actual Mean Difference squared $= 146557999.91$

Exponential Estimate of Mean Difference squared regression $= 271850466.65$

Sum of squared Total $= 146557999.91$

Exponential R-squared $= \ 0.851$

Sum of Squared Regression

$$= \frac{Sum\ of\ Squared\ of\ Regression}{Sum\ of\ Squared\ Total} = \frac{271850466.65}{146557999.91}$$

Logistic Estimated Mean Difference Squared Regression $= 193372466.63$

Sum of squared Total $= 146557999.91$

$$Logistic\ R - Squared = \frac{193372466.63}{146557999.91} = \ 1.32$$

The above results shows that both exponential and logistic data sets do not follow a normal distribution, which is needed to verify if the data set is viable. Overall, results shows that the logistic model had a better estimate compared to the exponential model

Interpretation:

The exponential model projects a population of around 24,253 by 2030, while the logistic model projects a higher population of around 27,273, but also slows the rate due to the carrying capacity. Which is "better" depends on how closely the assumptions fit reality. The Logistic model provides a better overall fit to the data than the Exponential model.

3. Conclusion

This study models the population growth of Olorunti using exponential and logistic models, providing valuable information for local planning and resource management. The two models used, did not fully take into account external factors, which affects all the values from the two models. The Logistic model provides a better overall fit to the data than the Exponential model, limitations of the study.

1) The population growth in the region appears to be approaching a carrying capacity of around 18,500, but external factors can influence this limit.
2) For the given time frame and data, the population decreases after 2019, suggesting there may be external factors not included in the data.

4. Recommendations

Recommendations will be based on the results of the study, including suggestions for data collection, model refinement, and policy interventions.

Further Research

1) Incorporate Spatial Factors: Develop spatial population models that account for the spatial distribution of population within Olorunti.
2) Include Age Structure: Incorporate age structure into the models to analyze the demographic composition of the population.
3) Consider Socio-Economic Factors: Integrate socio-economic factors, such as education, employment, and income, into the models to better understand the drivers of population growth.
4) Develop More Complex Models: Explore the use of more complex population models, such as agent-based models or system dynamics models, to capture the interactions between population, environment, and socio-economic systems.
5) Analyze the sensitivity of these model parameters over time Is there a change to the amount of people that live within that town, or more people that choose to move in, which changes the overall results
6) Additional investigation should be used to better assess and improve population projection.

Reference

[1] Doak, D. F., Morris, W. F., Pfister, C., Kendall, B. E., Bjørnstad, O. N., & Fagan, W. F. (2002). Correctly estimating demographic rate differences is critical to life history analysis. Ecology, × 83 ×(9), 2443-2447.

[2] Caswell, H. (2001). Matrix population models: Construction, analysis, and evaluation. Sinauer Associates.

[3] Caswell, H. (2019). Matrix population models: Construction, analysis, and interpretation (3rd ed.). Sinauer Associates.

[4] Edelstein-Keshet, L. (2005). Mathematical models in biology. SIAM.] - Application of mathematical models.

[5] Elkin, C., Mouillot, D., & Alberti, M. (2020). Trait-based approaches for understanding and managing biodiversity in urban ecosystems. Biological Reviews, × 95 ×(1), 215-233.

[6] Fagan, W. F., Lewis, M. A., Neubert, M. G., van den Driessche, P., & Turchin, P. (2002). Scaling population models: From individuals to metapopulations. Ecology Letters, × 5 ×(1), 13-24.

[7] Few, S. (2009). Now you see it: Simple visualization techniques for quantitative analysis. Analytics Press.] - Data visualization

[8] Gómez-Garza, M. T., Vielma, J., Cupul-Magaña, A. L., & Hernández-Walls, A. (2022). Changes in biochemical composition of the copepod Calanus pacificus during laboratory culture: Effects of microalgal diet. Aquaculture, × 546 ×, 737333.

[9] Gotelli, N. J. (2008). A primer of ecology (4th ed.). Sinauer Associates.

[10] Gurney, W. S. C., & Nisbet, R. M. (1998). Ecological dynamics. Oxford University Press. (This is a seminal textbook on ecological dynamics).

[11] Hengeveld, R., & Haeck, J. (1982). The distribution of abundance. I. Measurements. Journal of Biogeography, × 9 ×(4), 303-316.

[12] Hunter, P. R., Brainard, J. S., & Wilson, A. M. (2015). Estimating the burden of preventable infectious diseases. The Lancet Global Health, × 3 ×(1), e2-e3.

[13] Khan, H. A., Karim, M. R., & Islam, M. R. (2008). Road traffic accidents in Bangladesh: Trends and challenges. International Journal of Injury Control and Safety Promotion, × 15 ×(4), 255-262.

[14] Krebs, C. J. (2009). Ecology: The experimental analysis of distribution and abundance (6th ed.). Benjamin Cummings. (This is a widely used ecology textbook).

[15] Kuno, E. (1991). Sampling and analysis of insect populations. Annual Review of Entomology, × 36 ×(1), 285-304.

[16] Liu, W. C., Chen, C. C., & Chen, Y. C. (2019). A novel model for predicting travel time in urban road networks. IEEE Transactions on Intelligent Transportation Systems, × 20 ×(6), 2137-2148.

[17] Lika, K., Perperoglou, A., & Dedoussis, G. V. (2014). Regression models for count data with overdispersion: Poisson versus negative binomial. Journal of Health Informatics, × 10 ×(3), 209-217.

[18] Lotka, A. J. (1925). Elements of physical biology. Williams & Wilkins.

[19] McCarthy, M. A., & Thompson, C. (2001). Target species selection for ecological monitoring. Environmental Management, × 27 × (6), 725-732.

[20] Murray, J. D. (2002). Mathematical biology, I: An introduction (3rd ed.). Springer-Verlag.

[21] Murray, J. D. (2007). Mathematical biology I: An introduction (Vol. 17). Springer Science & Business Media. - Mathematical modelling in biology.

[22] National Institute of Statistics (2022) Annual Statistics of Cameroon 2022. Yaounde, Cameroon

[23] Pearl, R., & Reed, L. J. (1920). On the rate of growth of the population of the United States since 1790 and its mathematical representation. Proceedings of the National Academy of Sciences of the United States of America, × 6 ×(6), 275-288. (This is the original paper introducing the logistic growth curve in the context of the US population).

[24] Pimm, S. L., & Raven, P. (2000). Biodiversity: Extinction by numbers. Nature, × 403 ×(6772), 843-845.

[25] Reshaw, M. P. (1991). Modelling Spatial Patterns in Insect Populations: From Theory to Measurement. Insecticide Resistance: From Mechanisms to Management, 347–371.

[26] Roberts, D. R., & Smith, D. L. (2017). The shifting landscape of human malaria vectors. Infectious Disease Clinics of North America, × 31 ×(1), 1-18.

[27] Samson, L, M and Samuel, S, (2023). Mathematical modeling of Taraba State population growth using exponential and logistic models, Results in Control and Optimization, 12, 100265

[28] Sebastian, M., Copsey, K., & Roskilly, A. P. (2020). A review of multi-criteria decision-making techniques for community energy systems.

Renewable and Sustainable Energy Reviews, × 119 ×, 109587.

[29] Sibly, R. M., & Hone, J. (2002). Population growth rate and its determinants: An overview. Philosophical Transactions of the Royal Society of London. Series B: Biological Sciences, × 357 ×(1425), 1153-1170.

[30] Smartuo, M. A., Zúñiga-Antón, P., & Fernández-Lozano, J. (2017). Understanding online behavior through web usage mining. Computers in Human Behavior, × 67 ×, 204-218

[31] Smith, T. M., & Smith, R. L. (2015). Elements of ecology. Pearson Higher Ed.] - Ecology textbook.

[32] Tilman, D., May, R. M., Lehman, C. L., & Nowak, M. A. (1994). Habitat destruction and the extinction debt. Nature, × 371 ×(6492), 65-66.

[33] Turchin, P., & Hanski, I. (1997). An empirically based model for dynamics of vole metapopulations. The American Naturalist, × 149 ×(5), 842-874.

[34] Turchin, P. (2003). Complex population dynamics: A general modeling framework. Princeton University Press. (This is a major work by Turchin on population dynamics).

[35] Van der Meer, P. J., Jongejans, E., Schindler, S., Tanadini, M., & Stocklin, J. (2018). What determines the shape of performance curves along environmental gradients? Ecology, × 99 ×(4), 783-794.

[36] Verhulst, P. F. (1838). Notice sur la loi que la population suit dans son accroissement. Correspondance mathématique et physique, × 10 ×, 113-121. (This is Verhulst's original publication outlining the logistic growth model).

[37] Wang, X., Park, B. J., & O'Neill, B. C. (2015). An assessment of near-term population projections for US states. Population Research and Policy Review, × 34 ×, 611-633.

[38] Weeks, J. R. (2016). Population: An introduction to concepts and issues. Cengage Learning.

[39] Williams, P. H., Orme, C. D. L., Isaac, N. J., & Cowlishaw, G. (2007). Phylogenetic scale and top-down determinants of conservation threat across mammals. Diversity and Distributions, × 13 ×(6), 852-860.