

Operations on Pythagorean Neutrosophic Hypersoft Matrices with TOPSIS Method and their Application in Diagnosing Preterm Births

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Abstract

Introduction: Advanced mathematical models that can handle inaccurate and incomplete information are necessary for making decisions in uncertain circumstances. Pythagorean neutrosophic hypersoft matrices offer an adaptable structure for expressing uncertainty in multi-criteria issues. This research presents various operations on these matrices, demonstrates their essential characteristics through proofs, and utilizes them in medical decision-making by employing the max-min average composition and TOPSIS method to diagnose and rank preterm newborn patients.

Objectives: This research seeks to explore the operations and essential characteristics of Pythagorean neutrosophic hypersoft matrices (PNHSMs) and establish a strong decision-making model utilizing max–min average composition and the TOPSIS approach for assessing preterm infant conditions and prioritizing urgent cases.

Methods: A new multi-criteria decision-making method derived from max–min average composition has been introduced. Additionally, an evaluation matrix combined with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was created to evaluate the health status of preterm infants and prioritize patients based on their severity. A numerical example was provided to illustrate the real-world application of the suggested approach.

Results: The suggested framework effectively defined the theoretical characteristics of Pythagorean neutrosophic hypersoft matrices and showed their use in intricate decision-making issues. The combined max–min average composition and TOPSIS method efficiently assessed and prioritized preterm infants, facilitating precise recognition of the most urgent cases. The numerical example validated the effectiveness and practicality of the suggested algorithm.

Conclusions: This study presented operations on Pythagorean neutrosophic hypersoft matrices and demonstrated their characteristics. It additionally suggested a novel decision-making approach utilizing max-min average composition and TOPSIS. The findings indicated that the approach can accurately identify and prioritize critical preterm infants and may assist in decision-making under uncertainty.

Keywords: Pythagorean neutrosophic hypersoft matrix (PNHSM), Basic Operations, Max–Min average composition, Decision -making, Medical Diagnosis.

Introduction

Decisions with multiple possibilities can lead to unpredictability, which is frequently caused by complicated facts, a lack of expertise, or uncontrollable variables. To solve these issues, Molodstov [1] created the concept of soft sets, which are mathematical methods that handle unreliability without the need for parameterization. He used this strategy to generate a variety of results. In 2004, Maji et al. [2] broadened the SS (Soft Set) to IFSS (Intuitionistic fuzzy soft set) and they were tested for use in decision-making challenges. To tackle issues with decision-making, Shanmugasundaram et al. (2014) [3, 4] demonstrated a novel technique for intuitionistic fuzzy soft matrices. The school selection problem was also resolved by Dorthy and Sophia Porchelvi., (2020) [5] using IFSS.

Peng et al., (2015) [6] suggested an algorithm that use the PFSS (Pythagorean fuzzy soft set), along with a number of set operations. Furthermore, the PFSS was protracted to a PFSM (Pythagorean fuzzy soft matrix) by A. Guleria and R.K. Bajaj (2019) [7], who also defined a number of binary operations and used the method to address medical diagnosis issues. Naeem et al. (2021) [8] advocated using Pythagorean fuzzy generating operators, TOPSIS, and VIKOR approaches to deal with medical difficulties. The NSM (Neutrosophic Soft Matrix) was covered by Jafar, M.N., et. al., [9] (2020), who also used it to handle decision-making issues in the agricultural industry.

In 1998, Smarandache [10] presented NS (neutrosophic sets), probability, and logic to resolve ambivalence and obscurity. Ajay et al. (2022) [11] expanded this

structure to Pythagorean neutrosophic soft sets in order to address decision-making issues. M. Palanikumar and Arulmozhi [12] stated aggregation procedures for Pythagorean neutrosophic soft sets in 2022, utilizing the TOPSIS and VIKOR methods to address MCGDM (multi-criteria group decision-making) challenges. Hwang et al. (1981) [13] suggested MADM (multi-attribute decision-making) techniques to handle a variety of decision-making troubles.

A HSS (Hypersoft Set) was developed by Smarandache (2018) [14] to address issues brought on by numerous goals and features. This extension was designed to offer a solution for dealing with more uncertain settings. This idea was expanded upon by Zulqarnain et al. (2021) [15] to the PFHSS (Pythagorean fuzzy hypersoft set), which was established to use correlation coefficients to solve decision-making difficulties. Jafar et al. (2023) [16] expanded this structure to Pythagorean fuzzy hypersoft matrices to select treatment facilities for wastewater. Furthermore, Zulqarnain et. al., (2020, 2021) [17, 18] expanded this paradigm to the neutrosophic hypersoft set, presenting it in matrix form to solve decision-making issues. Additionally, in 2024, Jayasudha J and Raghavi S [19,20] formulated novel operations on neutrosophic hypersoft matrices and utilized them to making medical decisions issues. Using a neutrosophic framework to measure involution, evolution, and unpredictability in species, Paraskevas, A., and Madas, M. [21] 2024 developed a way to combine both common and unusual features in environmental adaption. Finally, Saeed et. al., (2024) [22] transformed neutrosophic hypersoft set notion into PNHSS (Pythagorean Neutrosophic Hypersoft Sets) to address the work-life balance issue. Ramya G. and Shalini F. A. [23] introduced the matrix form of the Pythagorean Neutrosophic Hypersoft Matrix (PNHSM) to tackle practical decision-making challenges. Matrices provide significant advantages in mathematics, making them essential for tackling a variety of issues. Inspired by these benefits, we expanded multiple operations on the PNHSM to tackle medical decision-making challenges.

Preterm newborns are more susceptible to health complications such as RDS (respiratory distress syndrome), infections, challenges in feeding, and Thermal regulation disorders. These individuals may endure developmental delays and face long-term cognitive and physical problems. Numerous preterm newborns require specialized care in a neonatal

intensive care unit (NICU) for careful monitoring and assistance. The earlier an infant is born, the higher the chance of difficulties, emphasizing the significance of close monitoring and appropriate measures for their well-being. To solve these difficulties and identify premature newborns, we offer an approach based on the Pythagorean Neutrosophic Hypersoft Matrix. In addition, we extend the TOPSIS approach for PNHSM to assist in the identification of critical preterm children.

This work provides the following contributions: In Section 2, we cover several fundamental ideas. Section 3 introduces the operations on Pythagorean Neutrosophic Hypersoft Matrix and illustrates them with appropriate examples. Section 4, we introduce a max-min average composition strategy and extend the TOPSIS method to PNHSM. Section 5 uses a numerical instance to highlight the use of this approach in detecting health issues in preterm newborns. In Section 6, we compare our suggested strategy to previous studies. Finally, Section 7 wraps up the paper.

Preliminaries

2.1. Definition [14]

Let Q represent a universal set, and $\mathcal{P}(Q)$ represent its power set. Let $\mathbb{B}^1, \mathbb{B}^2, \mathbb{B}^3, \dots, \mathbb{B}^n$ (where $n \geq 1$) as n well-defined attributes, with the associated attribute values being the set $\mathbb{B}^1, \mathbb{B}^2, \mathbb{B}^3, \dots, \mathbb{B}^n$, where $\mathbb{B}^i \cap \mathbb{B}^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$. A **HSS (Hypersoft Set)** over Q is the pair $(\mathcal{A}, \mathbb{B}^1 \times \mathbb{B}^2 \times \mathbb{B}^3 \dots \mathbb{B}^n)$, where $\mathcal{A}: \mathbb{B}^1 \times \mathbb{B}^2 \times \mathbb{B}^3 \dots \mathbb{B}^n \rightarrow \mathcal{P}(Q)$.

2.2. Definition [22]

Let Q represent a universal set, and $\mathcal{P}(Q)$ represent its power set. Let $\mathbb{B}^1, \mathbb{B}^2, \mathbb{B}^3, \dots, \mathbb{B}^n$ (where $n \geq 1$) as n well-defined attributes, with the associated attribute values being the set $\mathbb{B}^1, \mathbb{B}^2, \mathbb{B}^3, \dots, \mathbb{B}^n$, where $\mathbb{B}^i \cap \mathbb{B}^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$. A **PNHSS (Pythagorean Neutrosophic Hypersoft Set)** over Q is a pair $(\mathcal{A}, \mathbb{B}^1 \times \mathbb{B}^2 \times \mathbb{B}^3 \dots \mathbb{B}^n)$, where $\mathcal{A}: \mathbb{B}^1 \times \mathbb{B}^2 \times \mathbb{B}^3 \dots \mathbb{B}^n \rightarrow \mathcal{P}(Q)$ and is stated as $\mathcal{A}(\mathbb{B}^1 \times \mathbb{B}^2 \times \mathbb{B}^3 \dots \mathbb{B}^n) = \{ \langle o, T(o), I(o), F(o) \rangle, o \in Q \}$ where Truth and Falsity values are dependent $0 \leq T^2(o) + F^2(o) \leq 1$ and indeterminacy is independent value $0 \leq T^2(o) + I^2(o) + F^2(o) \leq 2$

2.3. Definition [23]

Assume $Q = \{q^1, q^2, \dots, q^\sigma\}$ is a universal set and $k = \{k_1, k_2, \dots, k_\omega\}$ represents a collection of distinct ω attributes, each having appropriate value for its

attribute of $k_1^a, k_2^b, \dots, k_\omega^z$ where $a, b, c, \dots, z = 1, 2, 3, \dots, n$. Define $\mathbb{P}(\varrho)$ as the powerset of ϱ . A PNHS over ϱ is defined as $(\mathbb{A}, k_1^a \times k_2^b \times \dots \times k_\omega^z)$ such that $(\mathbb{A}, k_1^a \times k_2^b \times \dots \times k_\omega^z) \rightarrow \mathbb{P}(\varrho)$, defined by $(\mathbb{A}, k_1^a \times k_2^b \times \dots \times k_\omega^z) = \{<$
 $o, T_i(o), I_i(o), F_i(o) > o \in \varrho, i \in k_1^a \times k_2^b \times \dots \times k_\omega^z\}$ where $0 \leq T_i^2(o) + I_i^2(o) + F_i^2(o) \leq 2$. In Table 1, a PNHSM is defined in the following matrix form:

Table 1: The PNHSM of PNHSs $(\mathbb{A}, k_1^a \times k_2^b \times \dots \times k_\omega^z)$

ϱ	k_1^a	k_2^b	$\dots$	k_ω^z
ϱ^1	$\mathbb{X}_{\varrho_u}(\varrho^1, k_1^a)$	$\mathbb{X}_{\varrho_u}(\varrho^1, k_2^b)$	$\dots$	$\mathbb{X}_{\varrho_u}(\varrho^1, k_\omega^z)$
ϱ^2	$\mathbb{X}_{\varrho_u}(\varrho^2, k_1^a)$	$\mathbb{X}_{\varrho_u}(\varrho^2, k_2^b)$	$\dots$	$\mathbb{X}_{\varrho_u}(\varrho^2, k_\omega^z)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ϱ^σ	$\mathbb{X}_{\varrho_u}(\varrho^\sigma, k_1^a)$	$\mathbb{X}_{\varrho_u}(\varrho^\sigma, k_2^b)$	$\dots$	$\mathbb{X}_{\varrho_u}(\varrho^\sigma, k_\omega^z)$

If $\tau_{uv} = \mathbb{X}_{\varrho_u}(\varrho^i, k_j^\ell)$, where $i = 1, 2, 3, \dots, \sigma, j = 1, 2, 3, \dots, \omega, \ell = a, b, c, \dots, z$, then the matrix could be stated as the following: $[\tau_{uv}]_{\sigma \times \omega} =$

$$\begin{pmatrix} \tau_{11} & \tau_{12} & \dots & \tau_{1\omega} \\ \tau_{21} & \tau_{22} & \dots & \tau_{2\omega} \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{\sigma 1} & \tau_{\sigma 2} & \dots & \tau_{\sigma \omega} \end{pmatrix}$$

If $\tau_{uv} = \{< o, T_i(o), I_i(o), F_i(o) > o \in \varrho, i \in k_1^a \times k_2^b \times \dots \times k_\omega^z\}$, where $T_i(o), I_i(o), F_i(o)$ symbolizes the membership, indeterminacy and non-membership of ϱ in the PNHSs respectively, then we define a matrix as $\mathcal{Q}_{\sigma \times \omega} =$

$$\begin{bmatrix} (T_{11}, I_{11}, F_{11}) & (T_{12}, I_{12}, F_{12}) & \dots & (T_{1\omega}, I_{1\omega}, F_{1\omega}) \\ (T_{21}, I_{21}, F_{21}) & (T_{22}, I_{22}, F_{22}) & \dots & (T_{2\omega}, I_{2\omega}, F_{2\omega}) \\ \vdots & \vdots & \ddots & \vdots \\ (T_{\sigma 1}, I_{\sigma 1}, F_{\sigma 1}) & (T_{\sigma 2}, I_{\sigma 2}, F_{\sigma 2}) & \dots & (T_{\sigma \omega}, I_{\sigma \omega}, F_{\sigma \omega}) \end{bmatrix}$$

This is a PNHSM of order $\sigma \times \omega$ over ϱ .

perations with Pythagorean Neutrosophic Hypersoft Matrices

Many computing algorithms rely on matrices to effectively address dimensionality issues in domains such as engineering, social sciences, and medical sciences. This part presents various operations on PNHSs, accompanied by example illustrations. The suggested operations are modified and expanded from those introduced by Ramya G., et al. [23].

Standard operations on two Pythagorean Neutrosophic Hypersoft matrices $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)]$ and $\mathcal{F} = [T_{ij}^{\mathcal{F}}(o), I_{ij}^{\mathcal{F}}(o), F_{ij}^{\mathcal{F}}(o)] \in \text{PNHSM}_{\sigma \times \omega}$ can be represented as follows:

1. Complement

If $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)]$ then $\mathcal{Y}^c = [F_{ij}^{\mathcal{Y}}(o), 1 - I_{ij}^{\mathcal{Y}}(o), T_{ij}^{\mathcal{Y}}(o)], \forall i \& j$.

2. Union

Let $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)], \mathcal{F} = [T_{ij}^{\mathcal{F}}(o), I_{ij}^{\mathcal{F}}(o), F_{ij}^{\mathcal{F}}(o)] \in \text{PNHSM}_{\sigma \times \omega}$ be the two PNHSM of order $\sigma \times \omega$. The union of $\mathcal{Y}$ and $\mathcal{F}$ is characterized by $\mathcal{Y} \cup \mathcal{F} =$

$$[\max(T_{ij}^{\mathcal{Y}}, T_{ij}^{\mathcal{F}}), \frac{I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}}}{2}, \min(F_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}})], \forall i \& j.$$

3. Intersection

Let $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)], \mathcal{F} = [T_{ij}^{\mathcal{F}}(o), I_{ij}^{\mathcal{F}}(o), F_{ij}^{\mathcal{F}}(o)] \in \text{PNHSM}_{\sigma \times \omega}$ be the two PNHSM of order $\sigma \times \omega$. The intersection of $\mathcal{Y}$ and $\mathcal{F}$ is characterized by

$$[\min(T_{ij}^{\mathcal{Y}}, T_{ij}^{\mathcal{F}}), \frac{I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}}}{2}, \max(F_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}})], \forall i \& j.$$

4. Addition

Let $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)], \mathcal{F} = [T_{ij}^{\mathcal{F}}(o), I_{ij}^{\mathcal{F}}(o), F_{ij}^{\mathcal{F}}(o)] \in \text{PNHSM}_{\sigma \times \omega}$ be the two PNHSM of order $\sigma \times \omega$. The addition of $\mathcal{Y}$ and $\mathcal{F}$ is characterized by $\mathcal{Y} + \mathcal{F} = [T_{ij}^{\mathcal{Y}} + T_{ij}^{\mathcal{F}} - T_{ij}^{\mathcal{Y}} \cdot T_{ij}^{\mathcal{F}}, I_{ij}^{\mathcal{Y}} \cdot I_{ij}^{\mathcal{F}}, F_{ij}^{\mathcal{Y}} \cdot F_{ij}^{\mathcal{F}}], \forall i \& j$.

5. Product

Let $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)], \mathcal{F} = [T_{ij}^{\mathcal{F}}(o), I_{ij}^{\mathcal{F}}(o), F_{ij}^{\mathcal{F}}(o)] \in \text{PNHSM}_{\sigma \times \omega}$ be the two PNHSM of order $\sigma \times \omega$. The product of $\mathcal{Y}$ and $\mathcal{F}$ is characterized by $\mathcal{Y} \cdot \mathcal{F} = [T_{ij}^{\mathcal{Y}} \cdot T_{ij}^{\mathcal{F}}, I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}} - I_{ij}^{\mathcal{Y}} \cdot I_{ij}^{\mathcal{F}}, F_{ij}^{\mathcal{Y}} + F_{ij}^{\mathcal{F}} - F_{ij}^{\mathcal{Y}} \cdot F_{ij}^{\mathcal{F}}], \forall i \& j$.

Example: 3.1

Let $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)], \mathcal{F} = [T_{ij}^{\mathcal{F}}(o), I_{ij}^{\mathcal{F}}(o), F_{ij}^{\mathcal{F}}(o)] \in \text{PNHSM}_{\sigma \times \omega}$ be the two PNHSM of order $\sigma \times \omega$. The results presented above can be developed as

$$\mathcal{Y} = \begin{bmatrix} (0.4, 0.6, 0.2) & (0.7, 0.2, 0.1) & (0.8, 0.1, 0.1) \\ (0.5, 0.7, 0.1) & (0.1, 0.6, 0.3) & (0.7, 0.5, 0.5) \\ (0.1, 0.3, 0.6) & (0.9, 0.1, 0.2) & (0.2, 0.7, 0.7) \end{bmatrix}$$

$$\mathcal{F} = \begin{bmatrix} (0.3,0.7,0.1) & (0.8,0.2,0.1) & (0.7,0.3,0.1) \\ (0.5,0.3,0.6) & (0.1,0.5,0.2) & (0.3,0.5,0.1) \\ (0.2,0.3,0.3) & (0.4,0.4,0.2) & (0.6,0.1,0.3) \end{bmatrix}$$

The preceding details are as follows

$$\mathcal{Y}^\circ = \begin{bmatrix} (0.2,0.4,0.4) & (0.1,0.8,0.7) & (0.1,0.9,0.8) \\ (0.1,0.3,0.5) & (0.3,0.4,0.1) & (0.5,0.5,0.7) \\ (0.6,0.7,0.1) & (0.2,0.9,0.9) & (0.7,0.3,0.2) \end{bmatrix}$$

$$\mathcal{Y} \cup \mathcal{F} = \begin{bmatrix} (0.4,0.65,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) \\ (0.5,0.5,0.1) & (0.1,0.55,0.2) & (0.7,0.5,0.1) \\ (0.2,0.3,0.3) & (0.9,0.25,0.2) & (0.6,0.4,0.3) \end{bmatrix}$$

$$\mathcal{Y} \cap \mathcal{F} = \begin{bmatrix} (0.3,0.65,0.2) & (0.7,0.2,0.1) & (0.7,0.2,0.1) \\ (0.5,0.5,0.6) & (0.1,0.55,0.3) & (0.3,0.5,0.5) \\ (0.1,0.3,0.6) & (0.4,0.25,0.2) & (0.2,0.4,0.7) \end{bmatrix}$$

$$\mathcal{Y} + \mathcal{F} = \begin{bmatrix} (0.58,0.42,0.02) & (0.94,0.04,0.01) & (0.94,0.03,0.01) \\ (0.75,0.21,0.06) & (0.19,0.3,0.06) & (0.79,0.25,0.05) \\ (0.28,0.09,0.18) & (0.94,0.04,0.04) & (0.68,0.07,0.21) \end{bmatrix}$$

$$\mathcal{Y} \cdot \mathcal{F} = \begin{bmatrix} (0.12,0.88,0.28) & (0.56,0.36,0.19) & (0.16,0.37,0.19) \\ (0.25,0.79,0.64) & (0.01,0.8,0.44) & (0.21,0.75,0.55) \\ (0.02,0.51,0.72) & (0.36,0.46,0.36) & (0.12,0.73,0.79) \end{bmatrix}$$

Proposition: 3.1

Let $\mathcal{Y}$ and $\mathcal{F} \in PNHSM_{\sigma \times \omega}$ be two PNHSMs. The subsequent outcomes hold:

1. $\mathcal{Y} \cup \mathcal{F} = \mathcal{F} \cup \mathcal{Y}$
2. $\mathcal{Y} \cap \mathcal{F} = \mathcal{F} \cap \mathcal{Y}$
3. $\mathcal{Y} + \mathcal{F} = \mathcal{F} + \mathcal{Y}$
4. $\mathcal{Y} \cdot \mathcal{F} = \mathcal{F} \cdot \mathcal{Y}$
5. $(\mathcal{Y}^\circ + \mathcal{F}^\circ)^\circ = \mathcal{Y} \cdot \mathcal{F}$
6. $(\mathcal{Y}^\circ \cdot \mathcal{F}^\circ)^\circ = \mathcal{Y} + \mathcal{F}$

Proof: Let $\mathcal{Y} = [T_{ij}^{\mathcal{Y}}(o), I_{ij}^{\mathcal{Y}}(o), F_{ij}^{\mathcal{Y}}(o)]$, $\mathcal{F} = [T_{ij}^{\mathcal{F}}(o), I_{ij}^{\mathcal{F}}(o), F_{ij}^{\mathcal{F}}(o)] \in PNHSM_{\sigma \times \omega}$. For every i & j we have,

$$\begin{aligned} \text{i. } \mathcal{Y} \cup \mathcal{F} &= [\max(T_{ij}^{\mathcal{Y}}, T_{ij}^{\mathcal{F}}), \frac{I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}}}{2}, \min(F_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}})] \\ &= [\max(T_{ij}^{\mathcal{F}}, T_{ij}^{\mathcal{Y}}), \frac{I_{ij}^{\mathcal{F}} + I_{ij}^{\mathcal{Y}}}{2}, \min(F_{ij}^{\mathcal{F}}, F_{ij}^{\mathcal{Y}})] \\ &= \mathcal{F} \cup \mathcal{Y}. \end{aligned}$$

$$\begin{aligned} \text{ii. } \mathcal{Y} \cap \mathcal{F} &= [\min(T_{ij}^{\mathcal{Y}}, T_{ij}^{\mathcal{F}}), \frac{I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}}}{2}, \max(F_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}})] \\ &= [\min(T_{ij}^{\mathcal{F}}, T_{ij}^{\mathcal{Y}}), \frac{I_{ij}^{\mathcal{F}} + I_{ij}^{\mathcal{Y}}}{2}, \max(F_{ij}^{\mathcal{F}}, F_{ij}^{\mathcal{Y}})] \\ &= \mathcal{F} \cap \mathcal{Y}. \end{aligned}$$

iii.

$$\mathcal{Y} + \mathcal{F} = [T_{ij}^{\mathcal{Y}} + T_{ij}^{\mathcal{F}} - T_{ij}^{\mathcal{Y}} \cdot T_{ij}^{\mathcal{F}}, I_{ij}^{\mathcal{Y}} \cdot I_{ij}^{\mathcal{F}}, F_{ij}^{\mathcal{Y}} \cdot F_{ij}^{\mathcal{F}}]$$

$$\begin{aligned} &= [T_{ij}^{\mathcal{F}} + T_{ij}^{\mathcal{Y}} - T_{ij}^{\mathcal{F}} \cdot T_{ij}^{\mathcal{Y}}, I_{ij}^{\mathcal{F}} \cdot I_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}} \cdot F_{ij}^{\mathcal{Y}}] \\ &= \mathcal{F} + \mathcal{Y}. \end{aligned}$$

$$\begin{aligned} \text{iv. } \mathcal{Y} \cdot \mathcal{F} &= [T_{ij}^{\mathcal{Y}} \cdot T_{ij}^{\mathcal{F}}, I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}} - I_{ij}^{\mathcal{Y}} \cdot I_{ij}^{\mathcal{F}}, F_{ij}^{\mathcal{Y}} + F_{ij}^{\mathcal{F}} - F_{ij}^{\mathcal{Y}} \cdot F_{ij}^{\mathcal{F}}] \\ &= [T_{ij}^{\mathcal{F}} \cdot T_{ij}^{\mathcal{Y}}, I_{ij}^{\mathcal{F}} + I_{ij}^{\mathcal{Y}} - I_{ij}^{\mathcal{F}} \cdot I_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}} + F_{ij}^{\mathcal{Y}} - F_{ij}^{\mathcal{F}} \cdot F_{ij}^{\mathcal{Y}}] \\ &= \mathcal{F} \cdot \mathcal{Y}. \end{aligned}$$

Similarly, the rest can be easily proven.

Proposition: 3.2

Consider $\mathcal{Y}, \mathcal{F}$ and $d \in PNHSM_{\sigma \times \omega}$ as three PNHSMs, the associativity of operations yields the following facts:

- (i) $(\mathcal{Y} \cup \mathcal{F}) \cup d = \mathcal{Y} \cup (\mathcal{F} \cup d)$
- (ii) $(\mathcal{Y} \cap \mathcal{F}) \cap d = \mathcal{Y} \cap (\mathcal{F} \cap d)$
- (iii) $(\mathcal{Y} + \mathcal{F}) + d = \mathcal{Y} + (\mathcal{F} + d)$
- (iv) $(\mathcal{Y} \cdot \mathcal{F}) \cdot d = \mathcal{Y} \cdot (\mathcal{F} \cdot d)$

Proof: For all i & j , it holds that,

$$\begin{aligned} \text{(i) } (\mathcal{Y} \cup \mathcal{F}) \cup d &= [\max(T_{ij}^{\mathcal{Y}}, T_{ij}^{\mathcal{F}}), \frac{I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}}}{2}, \min(F_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}})] \cup [(T_{ij}^d, I_{ij}^d, F_{ij}^d)] \\ &= [\{\max(T_{ij}^{\mathcal{Y}}, T_{ij}^{\mathcal{F}}), T_{ij}^d\}, \{\frac{I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}} + I_{ij}^d}{3}\}, \{\min(F_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}}), F_{ij}^d\}] \\ &= [\{\max(T_{ij}^{\mathcal{Y}}, T_{ij}^{\mathcal{F}}, T_{ij}^d)\}, \{\frac{I_{ij}^{\mathcal{Y}} + I_{ij}^{\mathcal{F}} + I_{ij}^d}{3}\}, \{\min(F_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{F}}, F_{ij}^d)\}] \\ &= [T_{ij}^{\mathcal{Y}}, I_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{Y}}] \cup [\{\max(T_{ij}^{\mathcal{F}}, T_{ij}^d)\}, \{\frac{I_{ij}^{\mathcal{F}} + I_{ij}^d}{2}\}, \{\min(F_{ij}^{\mathcal{F}}, F_{ij}^d)\}] \\ &= [T_{ij}^{\mathcal{Y}}, I_{ij}^{\mathcal{Y}}, F_{ij}^{\mathcal{Y}}] \cup [(T_{ij}^{\mathcal{F}}, I_{ij}^{\mathcal{F}}, F_{ij}^{\mathcal{F}}) \cup (T_{ij}^d, I_{ij}^d, F_{ij}^d)] \\ &= \mathcal{Y} \cup (\mathcal{F} \cup d). \end{aligned}$$

Remaining components were demonstrated in a similar manner.

Utilization of the PNHSM in decision-making processes

We offer the following definitions for max-min average composition, max-min composition, and evaluation matrix, based on an inclusive decision-making problem and the PNHSM notion.

Definition: 4.1

Let $\mathcal{Y} = [\mathcal{Y}_{ij}] \in PNHSM_{\sigma \times \alpha}$ and $\mathcal{F} = [\mathcal{F}_{jk}] \in PNHSM_{\alpha \times \omega}$ be two PNHSMs, then **max-min average composition** of PNHSMs is characterized by,
 $\mathcal{Y} \Phi \mathcal{F} = [\mathcal{D}_{ijk}]_{\sigma \times \omega} = [\max(\frac{T_{ij}^{\mathcal{Y}} + T_{jk}^{\mathcal{F}}}{2}), \min(\frac{I_{ij}^{\mathcal{Y}} + I_{jk}^{\mathcal{F}}}{2}), \min(\frac{F_{ij}^{\mathcal{Y}} + F_{jk}^{\mathcal{F}}}{2})] \forall i \& j$.

Definition: 4.2

Let $\mathcal{Y} = [\mathcal{Y}_{ij}] \in PNHSM_{\sigma \times \alpha}$ and $\mathcal{F} = [\mathcal{F}_{jk}] \in PNHSM_{\alpha \times \omega}$ be two PNHSMs, then **max-min composition** of PNHSMs is characterized by, $\mathcal{Y} * \mathcal{F} = [\mathcal{D}_{ijk}]_{\sigma \times \omega} = [\max\{\min(T_{ij}^{\mathcal{Y}}, T_{jk}^{\mathcal{F}})\}, \min\{\max(I_{ij}^{\mathcal{Y}}, I_{jk}^{\mathcal{F}})\}, \min\{\max(F_{ij}^{\mathcal{Y}}, F_{jk}^{\mathcal{F}})\}] \forall i \& j$.

Definition: 4.3

If $\mathcal{Y} = [\mathcal{Y}_{ij}] \in PNHSM_{\sigma \times \alpha}$ and $\mathcal{F} = [\mathcal{F}_{jk}] \in PNHSM_{\alpha \times \omega}$ and $\mathcal{Y}^{\circ}, \mathcal{F}^{\circ}$ are the complement of $\mathcal{Y}$ and $\mathcal{F}$ then the **evaluation matrix** is characterized by $\mathbb{W}(\mathcal{Y}, \mathcal{F}) = \frac{m+z}{2}$, where m is the matrix stated as $m = [(T_{ij}^{\mathcal{Y}} \Phi \mathcal{F}_{jk})^2 - (I_{ij}^{\mathcal{Y}} \Phi \mathcal{F}_{jk})^2 - (F_{ij}^{\mathcal{Y}} \Phi \mathcal{F}_{jk})^2]$ and z is the matrix stated as $z = [(T_{ij}^{\mathcal{Y}^{\circ}} \Phi \mathcal{F}_{jk}^{\circ})^2 - (I_{ij}^{\mathcal{Y}^{\circ}} \Phi \mathcal{F}_{jk}^{\circ})^2 - (F_{ij}^{\mathcal{Y}^{\circ}} \Phi \mathcal{F}_{jk}^{\circ})^2]$.

Algorithm-I

To overcome the problem, we propose an MCDM (multi-criteria decision-making) technique using a max-min average composition and evaluation matrix. Consider the improved PNHSS decision-making dilemma described below; [4] provides a thorough analysis of this approach.

Step (i): Create the PNHSs $(\mathcal{M}_a, \mathcal{S}_y)$, $(\mathcal{N}_b, \mathcal{C}_p)$ and calculate the PNHSMs $\mathcal{M}_a, \mathcal{N}_b$ that correspond with $(\mathcal{M}_a, \mathcal{S}_y)$ & $(\mathcal{N}_b, \mathcal{C}_p)$ respectively.

Step (ii): Determine the complement matrices of the PNHSMs $\mathcal{M}_a^{\circ}$ & $\mathcal{N}_b^{\circ}$.

Step (iii): Obtain PNHS max – min average composition $\mathcal{M}_a \Phi \mathcal{N}_b$ and $\mathcal{M}_a^{\circ} \Phi \mathcal{N}_b^{\circ}$.

Step (iv): Using definition 4.3, compute matrices m, z to produce the evaluation matrix $s(\mathcal{M}_a \Phi \mathcal{N}_b, \mathcal{M}_a^{\circ} \Phi \mathcal{N}_b^{\circ})$.

Step (v): Find the highest score $\mathbb{S}_{ij}$, for all patient $\mathbb{I}_i$. We therefore conclude that the patient $\mathbb{I}_i$ has disease $\mathbb{D}_i$.

Algorithm-II (PNHS TOPSIS Method)

TOPSIS is well-suited to dealing with similarity-based difficulties, such as symptom comparisons, so we apply it to PNHSM to solve medical assessment decision-making challenges. A detailed description of TOPSIS can be provided in [12]. The procedures for using the PNHS-TOPSIS approach to assess each patient's severe state are detailed below:

Step (i): Create a weighted characteristic matrix g by utilizing ranking scores derived in Step (iv) of Algorithm I. These results are then matched to the relevant linguistic analysis from Table 2.

$$g = \begin{bmatrix} \mathbb{G}_{11} & \mathbb{G}_{12} & \cdots & \mathbb{G}_{1n} \\ \mathbb{G}_{21} & \mathbb{G}_{22} & \cdots & \mathbb{G}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{G}_{i1} & \mathbb{G}_{i2} & \cdots & \mathbb{G}_{in} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{G}_{l1} & \mathbb{G}_{l2} & \cdots & \mathbb{G}_{ln} \end{bmatrix} = [\mathbb{G}_{ij}]_{l \times n}$$

Step (iii): Create the standardizing decision matrix $\mathcal{D}$, and compute each element d_{ij} as follows:

$$d_{ij} = \frac{\mathbb{G}_{ij}}{\sqrt{\sum_{k=1}^l \mathbb{G}_{ki}^2}} \quad (1)$$

$$\mathcal{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{i1} & d_{i2} & \cdots & d_{in} \\ \vdots & \vdots & \ddots & \vdots \\ d_{l1} & d_{l2} & \cdots & d_{ln} \end{bmatrix} = [d_{ij}]_{l \times n}$$

Step (iii): In this phase, the decision maker provides a collection of weights $W = \{w_1, w_2, \dots, w_j, \dots, w_n\}$, where $\sum_{j=1}^n w_j = 1$, which will be added to the evaluation matrix. The weighted component matrix η is created by multiplying every column of the standardizing decision matrix $\mathcal{D}$ with its appropriate weight w_j .

$$\eta = \begin{bmatrix} \eta_{11} & \eta_{12} & \cdots & \eta_{1n} \\ \eta_{21} & \eta_{22} & \cdots & \eta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{i1} & \eta_{i2} & \cdots & \eta_{in} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{l1} & \eta_{l2} & \cdots & \eta_{ln} \end{bmatrix} = [\eta_{ij}]_{l \times n},$$

where $\eta_{ij} = w_j d_{ij}$. (2)

Step (iv): Use the formulae below to find Positive and Negative Ideal solution ($\mathbb{PIS}$) & ($\mathbb{NIS}$):

$$\mathbb{PIS} = \{\eta_1^+, \eta_2^+, \dots, \eta_j^+, \dots, \eta_n^+\} = \{\max_{i \in 1, 2, \dots, l} (\eta_{ij})\} \quad (3)$$

$$\mathbb{NIS} = \{\eta_1^-, \eta_2^-, \dots, \eta_j^-, \dots, \eta_n^-\} = \{\min_{i \in 1, 2, \dots, l} (\eta_{ij})\} \quad (4)$$

Step (v): Using the formulas provided below, compute the distance measures for the PIS ($\mathfrak{B}_i^+$) and NIS ($\mathfrak{B}_i^-$) according to each variable:

$$\mathfrak{B}_i^+ = \sqrt{\sum_{j=1}^n (\eta_{ij} - \eta_j^+)^2}, \forall i \in 1, 2, \dots, l \quad (5)$$

$$\mathfrak{B}_i^- = \sqrt{\sum_{j=1}^n (\eta_{ij} - \eta_j^-)^2}, \forall i \in 1, 2, \dots, l \quad (6)$$

Step (vi): Using the calculation below, determine the relative closeness of each alternative

$$\chi_i^+ = \frac{\eta_i^-}{\eta_i^- + \eta_i^+}, 0 \leq \chi_i^+ \leq 1, \forall i \in 1, 2, \dots, l \quad (7)$$

Step: (vii) List the alternatives according to their preferred order.

Preterm birth Infants Diagnostic via PNHSM using TOPSIS

This lesson explains the preterm delivery, its symptoms, and the health complications for premature babies. We use TOPSIS method to evaluate and diagnose key health issues in at-risk newborns.

Case Study

A preterm baby is one who is born too soon, typically three weeks earlier the scheduled date. Full – term gestation (foetal development) lasts around forty weeks. Preterm birth happens when a baby arrives at 37 weeks of gestation or less.

Symptoms

Preterm newborns may experience a number of obstacles as a result of their early arrival. Their organs and systems may be underdeveloped, resulting in health issues. The symptoms and indicators of preterm babies vary depending on how early they are born and the severity of their early years.

Presented below are some frequent symptoms and characteristics of preterm babies:

S₁: Underweight at birth: Underweight at birth denotes to newborns that are born weighing less than 5 pounds (2,500g). A typical newborn weighs around 8 pounds. Despite their small size, a baby with a low weight at birth can be healthy. However, a baby with a low birth weight might suffer from a variety of major health issues.

S₂: Difficulties with breathing: Newborns, particularly those born earlier 34 weeks, may have difficulty

breathing since their lungs are not completely developed. Some people may require breathing assistance such as oxygen or ventilatory support.

S₃: Apnea and bradycardia (Heart rate and breathing issues): Apnea (breathing pauses) and Bradycardia (slow heart rate) are frequent in premature infants, especially those birth prior twenty-eight weeks gestation. These issues often resolve as the baby develops and matures, but in certain circumstances, they may necessitate medical attention.

Complications

Here is an outline of the typical issues faced by preterm infants:

©₁: Respiratory issues:

Respiratory Distress Syndrome (RDS): One of the most prevalent problems in preterm infants, particularly those delivered before 34 weeks. RDS develops when the lungs of a newborn are immature and lack sufficient surfactant – a chemical that causes the tiny air-filled sacs in the lungs to open. The lungs collapse in the absence of adequate surfactant, causing difficulty breathing.

Bronchopulmonary Dysplasia (BPD): Premature newborns, especially those who require breathing machines or oxygen supply for an extended amount of time, are at risk for BPD. This illness is characterized by lung tissue scarring and can result in long-term breathing issues.

©₂: Cardiovascular issues:

Patent Ductus Arteriosus (PDA): This refers to a disorder in which the ductus arteriosus, which connects the pulmonary artery to the aorta, does not seal properly after birth. This is frequent in premature neonates and can lead to difficulties with blood circulation and oxygen delivery.

©₃: hearing impairment: Premature infants, particularly those who underwent prolonged oxygen treatment or mechanical breathing, have an increased risk of losing their hearing. This could be caused to injury to the eardrum or the pathways of nerves that carry seem to the brain.

©₄: Jaundice: Premature newborns frequently get neonatal jaundice. It develops when bilirubin (a toxic byproduct from the decomposition of the RBC)

accumulates in the baby's bloodstream. Premature newborns' livers might not be sufficiently developed to handle bilirubin adequately, resulting in a yellowish tinge to their skin and eyes.

Numerical Example

Let $\mathcal{H} = \{\mathbb{h}_1, \mathbb{h}_2, \mathbb{h}_3, \mathbb{h}_4\}$ represent the set of preterm newborns (patients) in a hospital. These newborns have the following symptoms (attributes): $\mathbb{S}_y = \{\mathbb{s}_{y_1}, \mathbb{s}_{y_2}, \mathbb{s}_{y_3}, \mathbb{s}_{y_4}\}$ including breathing difficulty, slow-moving heart rate, low birth weight, and yellowing of the skin and eyes.

The doctor's responsibility is to identify the patient who is exhibiting the most severe disease signs.

Consequently, the above-described attributes have been thoroughly categorized and can be grouped in the ways listed below:

$\mathbb{s}_{y_1}^a = \{\text{Very low, mild, High, Very High}\}$, $\mathbb{s}_{y_2}^b = \{<80 \text{ beatsP/min.}, 80-90 \text{ beatsP/min.}, 90-110 \text{ beatsP/min.}\}$,

$\mathbb{s}_{y_3}^c = \{<1500 \text{ grams, } 1500-2000\text{grams, } 2000-2400\text{grams}\}$,

$\mathbb{s}_{y_4}^d = \{\text{low tint of yellow, moderate yellowish tint, extreme yellowish tint}\}$.

Step (i):

PNHSS $(\mathcal{M}_a, \mathbb{S}_y)$ is defined as $\mathcal{M}_a: \mathbb{s}_{y_1}^a \times \mathbb{s}_{y_2}^b \times \mathbb{s}_{y_3}^c \times \mathbb{s}_{y_4}^d \rightarrow \mathbb{P}(\mathcal{H})$.

Assuming that $\mathcal{M}_a(\mathbb{s}_{y_1}^a \times \mathbb{s}_{y_2}^b \times \mathbb{s}_{y_3}^c \times \mathbb{s}_{y_4}^d) = \mathcal{M}_a(\text{Very high, } < 80 \text{ beatsP/min.}, < 1500\text{grams.}, \text{extreme yellowish tint})$ is the doctor's criteria. Four preterm newborns (patients) $\mathbb{h}_1, \mathbb{h}_2, \mathbb{h}_3, \mathbb{h}_4$ have been nominated based on the criteria listed above.

The severity of the symptoms for these four preterm newborns (patients) is shown in table 2.

Table 2: PNHSS for patients and symptoms

Patients↓/ Symptoms→	$\mathbb{s}_{y_1}$	$\mathbb{s}_{y_2}$	$\mathbb{s}_{y_3}$	$\mathbb{s}_{y_4}$
$\mathbb{h}_1$	(0.8,0.2,0.1)	(0.5,0.2,0.1)	(0.7,0.5,0.3)	(0.6,0.5,0.2)
$\mathbb{h}_2$	(0.5,0.2,0.6)	(0.6,0.1,0.5)	(0.5,0.2,0.1)	(0.5,0.2,0.1)
$\mathbb{h}_3$	(0.7,0.6,0.1)	(0.5,0.6,0.2)	(0.3,0.4,0.7)	(0.2,0.6,0.8)
$\mathbb{h}_4$	(0.1,0.3,0.5)	(0.8,0.7,0.3)	(0.5,0.3,0.1)	(0.3,0.4,0.6)

We define this PNHSS as a PNHSM by the following:

$$\mathcal{M}_a = \begin{matrix} & \mathbb{s}_{y_1} & \mathbb{s}_{y_2} & \mathbb{s}_{y_3} & \mathbb{s}_{y_4} \\ \begin{matrix} \mathbb{h}_1 \\ \mathbb{h}_2 \\ \mathbb{h}_3 \\ \mathbb{h}_4 \end{matrix} & \begin{bmatrix} (0.8,0.2,0.1) & (0.5,0.2,0.1) & (0.7,0.5,0.3) & (0.6,0.5,0.2) \\ (0.5,0.2,0.6) & (0.6,0.1,0.5) & (0.5,0.2,0.1) & (0.5,0.2,0.1) \\ (0.7,0.6,0.1) & (0.5,0.6,0.2) & (0.3,0.4,0.7) & (0.2,0.6,0.8) \\ (0.1,0.3,0.5) & (0.8,0.7,0.3) & (0.5,0.3,0.1) & (0.3,0.4,0.6) \end{bmatrix} \end{matrix}$$

The doctor can determine the complications for each newborn based on the severity of symptoms in the four preterm babies. Their symptoms are also linked to the following complications: $\mathbb{C}_{p_1}$ = Jaundice, $\mathbb{C}_{p_2}$ = Temperature instability, $\mathbb{C}_{p_3}$ = Respiratory problems, $\mathbb{C}_{p_4}$ = Increased risk of infections, $\mathbb{C}_{p_5}$ = Cardiac issues.

Consider PNHSS $(\mathcal{N}_b, \mathbb{C}_p)$ over $\mathbb{S}_y$, which represents the complications of their symptoms, and it's determined in Table 3.

Table: 3 PNHSS for symptoms and complications

Symptoms↓/ Complications→	$\mathbb{C}_{\mathcal{P}_1}$	$\mathbb{C}_{\mathcal{P}_2}$	$\mathbb{C}_{\mathcal{P}_3}$	$\mathbb{C}_{\mathcal{P}_4}$	$\mathbb{C}_{\mathcal{P}_5}$
$\mathbb{S}_{y_1}$	(0.5,0.3,0.8)	(0.7,0.5,0.6)	(0.8,0.7,0.3)	(0.5,0.5,0.8)	(0.7,0.3,0.1)
$\mathbb{S}_{y_2}$	(0.8,0.2,0.1)	(0.6,0.3,0.5)	(0.6,0.5,0.3)	(0.6,0.4,0.3)	(0.5,0.7,0.4)
$\mathbb{S}_{y_3}$	(0.5,0.6,0.6)	(0.7,0.6,0.5)	(0.5,0.4,0.3)	(0.6,0.5,0.2)	(0.2,0.5,0.8)
$\mathbb{S}_{y_4}$	(0.6,0.7,0.3)	(0.5,0.6,0.7)	(0.8,0.6,0.3)	(0.2,0.5,0.7)	(0.4,0.6,0.5)

We define this PNHS as a PNHSM by the following:

$$\mathcal{N}_{\mathcal{S}} = \begin{matrix} & \mathbb{C}_{\mathcal{P}_1} & \mathbb{C}_{\mathcal{P}_2} & \mathbb{C}_{\mathcal{P}_3} & \mathbb{C}_{\mathcal{P}_4} & \mathbb{C}_{\mathcal{P}_5} \\ \begin{matrix} \mathbb{S}_{y_1} \\ \mathbb{S}_{y_2} \\ \mathbb{S}_{y_3} \\ \mathbb{S}_{y_4} \end{matrix} & \begin{bmatrix} (0.5,0.3,0.8) & (0.7,0.5,0.6) & (0.8,0.7,0.3) & (0.5,0.5,0.8) & (0.7,0.3,0.1) \\ (0.8,0.2,0.1) & (0.6,0.3,0.5) & (0.6,0.5,0.3) & (0.6,0.4,0.3) & (0.5,0.7,0.4) \\ (0.5,0.6,0.6) & (0.7,0.6,0.5) & (0.5,0.4,0.3) & (0.6,0.5,0.2) & (0.2,0.5,0.8) \\ (0.6,0.7,0.3) & (0.5,0.6,0.7) & (0.8,0.6,0.3) & (0.2,0.5,0.7) & (0.4,0.6,0.5) \end{bmatrix} \end{matrix}$$

Step:(ii) The complements of the PNHSM derived in step (i) are determined below:

$$\mathcal{M}_a^{\circ} = \begin{matrix} & \mathbb{S}_{y_1} & \mathbb{S}_{y_2} & \mathbb{S}_{y_3} & \mathbb{S}_{y_4} \\ \begin{matrix} \mathbb{h}_1 \\ \mathbb{h}_2 \\ \mathbb{h}_3 \\ \mathbb{h}_4 \end{matrix} & \begin{bmatrix} (0.1,0.8,0.8) & (0.1,0.8,0.5) & (0.3,0.5,0.7) & (0.2,0.5,0.6) \\ (0.6,0.8,0.5) & (0.5,0.9,0.6) & (0.1,0.8,0.5) & (0.1,0.8,0.5) \\ (0.1,0.4,0.7) & (0.2,0.4,0.5) & (0.7,0.6,0.3) & (0.8,0.4,0.2) \\ (0.5,0.7,0.1) & (0.3,0.3,0.8) & (0.1,0.7,0.5) & (0.6,0.6,0.3) \end{bmatrix} \end{matrix}$$

$$\mathcal{N}_{\mathcal{S}}^{\circ} = \begin{matrix} & \mathbb{C}_{\mathcal{P}_1} & \mathbb{C}_{\mathcal{P}_2} & \mathbb{C}_{\mathcal{P}_3} & \mathbb{C}_{\mathcal{P}_4} & \mathbb{C}_{\mathcal{P}_5} \\ \begin{matrix} \mathbb{S}_{y_1} \\ \mathbb{S}_{y_2} \\ \mathbb{S}_{y_3} \\ \mathbb{S}_{y_4} \end{matrix} & \begin{bmatrix} (0.8,0.7,0.5) & (0.6,0.5,0.7) & (0.3,0.3,0.8) & (0.8,0.5,0.5) & (0.1,0.7,0.7) \\ (0.1,0.8,0.8) & (0.5,0.7,0.6) & (0.3,0.5,0.6) & (0.3,0.6,0.6) & (0.4,0.3,0.5) \\ (0.6,0.4,0.5) & (0.5,0.4,0.7) & (0.3,0.6,0.5) & (0.2,0.5,0.6) & (0.8,0.5,0.2) \\ (0.3,0.3,0.6) & (0.7,0.4,0.5) & (0.3,0.4,0.8) & (0.7,0.5,0.2) & (0.5,0.4,0.4) \end{bmatrix} \end{matrix}$$

Step: (iii) The maximum and minimum average compositions of the PNHSMs determined in steps (i) and (ii) were acquired as follows:

$$\mathcal{Y} = \mathcal{M}_a \Phi \mathcal{N}_{\mathcal{S}} = \begin{matrix} & \mathbb{C}_{\mathcal{P}_1} & \mathbb{C}_{\mathcal{P}_2} & \mathbb{C}_{\mathcal{P}_3} & \mathbb{C}_{\mathcal{P}_4} & \mathbb{C}_{\mathcal{P}_5} \\ \begin{matrix} \mathbb{h}_1 \\ \mathbb{h}_2 \\ \mathbb{h}_3 \\ \mathbb{h}_4 \end{matrix} & \begin{bmatrix} (0.65,0.2,0.1) & (0.75,0.25,0.3) & (0.8,0.3,0.2) & (0.65,0.3,0.2) & (0.75,0.25,0.1) \\ (0.7,0.15,0.2) & (0.6,0.2,0.3) & (0.65,0.3,0.2) & (0.6,0.25,0.15) & (0.6,0.25,0.3) \\ (0.65,0.4,0.65) & (0.7,0.45,0.35) & (0.75,0.4,0.2) & (0.6,0.45,0.25) & (0.7,0.45,0.1) \\ (0.8,0.3,0.2) & (0.7,0.4,0.3) & (0.7,0.35,0.2) & (0.7,0.4,0.15) & (0.65,0.3,0.3) \end{bmatrix} \end{matrix}$$

$$\mathcal{f} = \mathcal{M}_a^{\circ} \Phi \mathcal{N}_{\mathcal{S}}^{\circ} = \begin{matrix} & \mathbb{C}_{\mathcal{P}_1} & \mathbb{C}_{\mathcal{P}_2} & \mathbb{C}_{\mathcal{P}_3} & \mathbb{C}_{\mathcal{P}_4} & \mathbb{C}_{\mathcal{P}_5} \\ \begin{matrix} \mathbb{h}_1 \\ \mathbb{h}_2 \\ \mathbb{h}_3 \\ \mathbb{h}_4 \end{matrix} & \begin{bmatrix} (0.45,0.4,0.6) & (0.45,0.45,0.55) & (0.3,0.45,0.5) & (0.45,0.5,0.4) & (0.55,0.45,0.45) \\ (0.7,0.55,0.5) & (0.6,0.6,0.5) & (0.45,0.55,0.5) & (0.7,0.65,0.35) & (0.45,0.6,0.35) \\ (0.65,0.35,0.4) & (0.75,0.4,0.35) & (0.55,0.35,0.4) & (0.75,0.45,0.2) & (0.75,0.35,0.25) \\ (0.65,0.45,0.3) & (0.65,0.5,0.4) & (0.45,0.4,0.45) & (0.65,0.45,0.25) & (0.55,0.3,0.35) \end{bmatrix} \end{matrix}$$

Step: (iv) The evaluation matrices for the PNHSM $\mathcal{Y}$ and $\mathcal{f}$ computed in step (iii) are:

$$\mathcal{S}_{\mathcal{Y}} = \begin{matrix} & \mathbb{C}_{\mathcal{P}_1} & \mathbb{C}_{\mathcal{P}_2} & \mathbb{C}_{\mathcal{P}_3} & \mathbb{C}_{\mathcal{P}_4} & \mathbb{C}_{\mathcal{P}_5} \\ \begin{matrix} \mathbb{h}_1 \\ \mathbb{h}_2 \\ \mathbb{h}_3 \\ \mathbb{h}_4 \end{matrix} & \begin{bmatrix} -0.0975 & 0.0575 & 0.1875 & 0.0125 & 0.1575 \\ -0.0625 & -0.25 & -0.13 & -0.185 & -0.1225 \\ 0.14 & 0.2075 & 0.28 & 0.1175 & 0.305 \\ 0.3475 & 0.08 & 0.1275 & 0.225 & 0.21 \end{bmatrix} \end{matrix}$$

$$\mathcal{S}_{\mathcal{f}} = \begin{matrix} & \mathbb{C}_{\mathcal{P}_1} & \mathbb{C}_{\mathcal{P}_2} & \mathbb{C}_{\mathcal{P}_3} & \mathbb{C}_{\mathcal{P}_4} & \mathbb{C}_{\mathcal{P}_5} \end{matrix}$$

$$\begin{matrix} \mathbb{h}_1 \\ \mathbb{h}_2 \\ \mathbb{h}_3 \\ \mathbb{h}_4 \end{matrix} \begin{bmatrix} 0.1525 & 0.05 & -0.04 & 0.0725 & 0.23 \\ 0.4275 & 0.23 & 0.0725 & 0.405 & 0.05 \\ 0.24 & 0.2375 & 0.1025 & 0.2975 & 0.35 \\ 0.2925 & 0.1725 & 0.04 & 0.24 & 0.1225 \end{bmatrix}$$

The overall evaluation matrix of the evaluation matrices $\mathbb{s}_y$ & $\mathbb{s}_f$ determined above is:

$$\mathbb{U}(\mathcal{Y}, \mathcal{F}) =$$

$$\begin{matrix} & \mathbb{C}_{\mathcal{P}_1} & \mathbb{C}_{\mathcal{P}_2} & \mathbb{C}_{\mathcal{P}_3} & \mathbb{C}_{\mathcal{P}_4} & \mathbb{C}_{\mathcal{P}_5} \end{matrix}$$

$$\begin{matrix} \mathbb{h}_1 \\ \mathbb{h}_2 \\ \mathbb{h}_3 \\ \mathbb{h}_4 \end{matrix} \begin{bmatrix} 0.0275 & 0.05375 & 0.07375 & 0.0425 & \mathbf{0.19375} \\ \mathbf{0.1825} & -0.01 & -0.02875 & 0.11 & -0.03625 \\ 0.19 & 0.2225 & 0.19125 & 0.2075 & \mathbf{0.3275} \\ \mathbf{0.32} & 0.12625 & 0.08375 & 0.2325 & 0.16625 \end{bmatrix}$$

Step: (v) As a result, depending on the values of each of the components of the overall evaluation matrix produced in step (iv), patients $\mathbb{h}_1, \mathbb{h}_3$ are suffer from cardiac issues ($\mathbb{C}_{\mathcal{P}_5}$), while patients $\mathbb{h}_2, \mathbb{h}_4$ are suffer from jaundice ($\mathbb{C}_{\mathcal{P}_1}$).

To determine the severity of the disease in patients, we use the PNHS TOPSIS approach, as described in Algorithm II. Initially, we create grading values utilizing the linguistic terms outlined in Table 4.

Table 4: Linguistic principles for evaluating parameters.

Linguistic concepts	Evaluation criteria	Grading scale
Highly Reliable (HR)	0 – 0.05	4
Reliable (R)	0.06 – 0.1	3
Severe (S)	0.11 – 0.25	2
Crucial (C)	0.26 – 0.3	1
Very Crucial (VC)	0.31 – 0.5	0

Using Step (iv) and Table 4, we create a weighted component matrix in the following way:

$$\mathbb{g} = \begin{bmatrix} HR & HR & R & HR & S \\ S & HR & HR & S & HR \\ S & S & S & S & C \\ C & S & HR & S & S \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 3 & 4 & 2 \\ 2 & 4 & 4 & 2 & 4 \\ 2 & 2 & 2 & 2 & 1 \\ 1 & 2 & 4 & 2 & 2 \end{bmatrix}$$

Using Equation 1, create a standardizing decision matrix $\mathbb{D}$.

$$\mathbb{D} = \begin{bmatrix} 0.51 & 0.51 & 0.38 & 0.51 & 0.26 \\ 0.27 & 0.53 & 0.53 & 0.27 & 0.53 \\ 0.49 & 0.49 & 0.49 & 0.49 & 0.24 \\ 0.19 & 0.37 & 0.74 & 0.37 & 0.37 \end{bmatrix}$$

Assume the decision maker assigns relative weight to qualities like $W = \{w_1, w_2, w_3, w_4, w_5\} = (0.28, 0.3, 0.1, 0.05, 0.27)$. The weighted component matrix is then

$$\eta = \begin{bmatrix} 0.143 & 0.153 & 0.038 & 0.026 & 0.07 \\ 0.076 & 0.159 & 0.053 & 0.014 & 0.143 \\ 0.137 & 0.147 & 0.049 & 0.025 & 0.065 \\ 0.053 & 0.111 & 0.074 & 0.019 & 0.099 \end{bmatrix}$$

The PIS & NIS by utilizing the equations 3 & 4 as

$$\text{PIS} = \{0.143, 0.159, 0.074, 0.026, 0.143\}$$

$$\text{NIS} = \{0.053, 0.111, 0.038, 0.014, 0.065\}$$

Calculate the separation measures of PIS & NIS $\mathfrak{B}_i^+ = \sqrt{\sum_{j=1}^5 (\eta_{ij} - \eta_j^+)^2}$, $\mathfrak{B}_1^+ = 0.08$, $\mathfrak{B}_2^+ = 0.071$, $\mathfrak{B}_3^+ = 0.083$, $\mathfrak{B}_4^+ = 0.11$.

$$\mathfrak{B}_i^- = \sqrt{\sum_{j=1}^5 (\eta_{ij} - \eta_j^-)^2}$$
, $\mathfrak{B}_1^- = 0.1$, $\mathfrak{B}_2^- = 0.28$, $\mathfrak{B}_3^- = 0.093$, $\mathfrak{B}_4^- = 0.05$.

Using equation 4.7, the closest options to the optimum answer are $\chi_i^+ = \frac{\eta_i^-}{\eta_i^- + \eta_i^+}$.

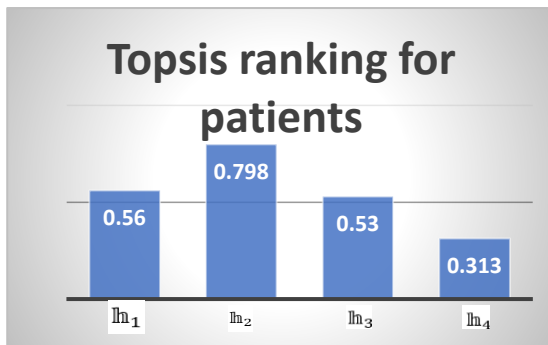
$$\chi_1^+ = 0.56, \chi_2^+ = 0.798, \chi_3^+ = 0.53, \chi_4^+ = 0.313.$$

Table 5 indicates that patient 2 ($\mathbb{h}_2$) has the most critical condition.

Table 5: Alternatives are ranked based on their relative proximity.

Relative Closeness	Ranking
$\chi_1^+ = 0.56$	2
$\chi_2^+ = 0.798$	1
$\chi_3^+ = 0.53$	3
$\chi_4^+ = 0.313$	4

Figure 1 shows a chart that illustrates the patient rating.



Results and Discussion

We have concluded that the suggested approach has wider application than existing methods based on our study and comparative analysis. When it comes to decision-making, it offers more thorough information to handle data ambiguity than current approaches. A lot of NS's mixed structures have also developed into particular PNHS situations with extra criteria. Matrices, which are well-known for their efficiency in handling huge datasets in a variety of computing domains, improve PNHS's capacity to represent data more accurately and empirically. Because of this, PNHS is a useful method for incorporating ambiguous and imprecise data into decisions. Using the integrated PNHS-TOPSIS approach, our recently developed algorithms are also very successful in detecting critical preterm birth baby problems. As per the outcomes of Algorithm I, the symptoms impacting each patient were recognized. Afterward, Algorithm II was employed to identify the most crucial patient by assessing the intensity of the recognized symptoms. Thus, compared to other hybrid neutrosophic set frameworks, our method is effective, simple, and flexible.

Conclusion

We have thoroughly defined the operations on the Pythagorean neutrosophic hypersoft matrix, as well as its numerous kinds and features. The suggested properties have been supported by reliable proofs. Furthermore, decision-making algorithms based on maximum and minimum average composition and evaluation matrices were used to diagnose health issues in preterm newborns using the TOPSIS approach, with a numerical example supplied for demonstration. We have also extended the TOPSIS approach for matrices to identify the most critical patients. This method gives a straightforward suggested answer for dealing with any form of difficult real-world problem. Future extensions of the suggested PNHS model may elaborate on the bipolar PNHS, distance and

similarity measures for PNHSs, and the VIKOR method within PNHSs.

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